Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Duisburg, den __________ ____________________________________

(Unterschrift der/des Studierenden)

Falls Klausurunterlagen vorzeitig abgegeben: _______________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Gesamtpunktzahl	
Angehobene Punktzahl	
%	
Bewertung gem. PO in Ziffern	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

____

(Datum und Unterschrift 2. Prüfer, Yan Liu, M. Eng.)

____

(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ____

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam “Control Engineering” is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**60**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

Problem 1 (15 Points)

1a) (4 Points)
A system features a PDT_2-transfer behavior with the parameters K, D, and ω_0. What are the calculated analytical and the practical initial and also the final values (2 x 2 cases) of its step response function for $D = 0$?

Calculative:
Initial value: 0
Final value: K

Practical:
Initial value $\to \infty$
Final value: not existent (permanent oscillation due to $D = 0$).

1b) (1 Point)
For a system the poles $s_{1,2} = -4$ (pole pair) and $s_{3,4} = \pm j100$ are given. Calculate the damping factor of each of the given poles. Simplify your result as far as possible.

$s_{1,2} = -4 \,:\, D = 1$
$s_{3,4} = \pm j100 \,:\, D = 0$

1c) (3 Points)

Refering to the given root-locus plot (with included pole-zero map) of an open loop system with negative feedback as shown in Figure 1.1, the system behavior of the resulting closed loop has to be determined.

Which value of the parameter $\tilde{K}$ defining the root locus gain features the best behavior of the closed loop system with respect to stability ($\tilde{K}=0$, small $\tilde{K}$, middle $\tilde{K}$, large $\tilde{K}$)? Draw in the root-locus plot the boundary values K_{krit1} and K_{krit2}. For which condition of K_{krit1} and K_{krit2} exists a stable operating range of the closed loop?

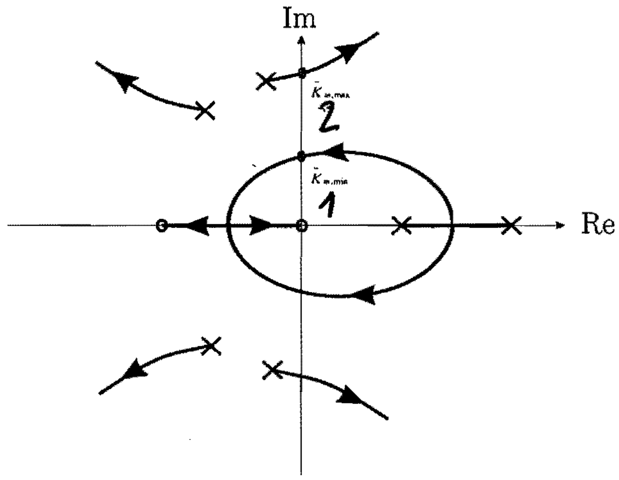


Figure 1.1: Root locus plot

The system has a stable behavior using 'middle $\tilde{K}$'.
Condition: $K_{\text{krit1}} \leq K_{\text{krit2}}$

$K_{\text{krit1}} \leq \tilde{K} \leq K_{\text{krit2}}$: stable behavior.

1d) (2 Points)

The nyquist plot of an open loop system is given in Figure 1.2. Indicate the gain margin as well as the phase margin in Figure 1.2. Additionally, determine the concrete values of the gain margin as well as the phase margin of this system. Is the closed loop of the given system asymptotically stable, stable, or unstable? State reasons.

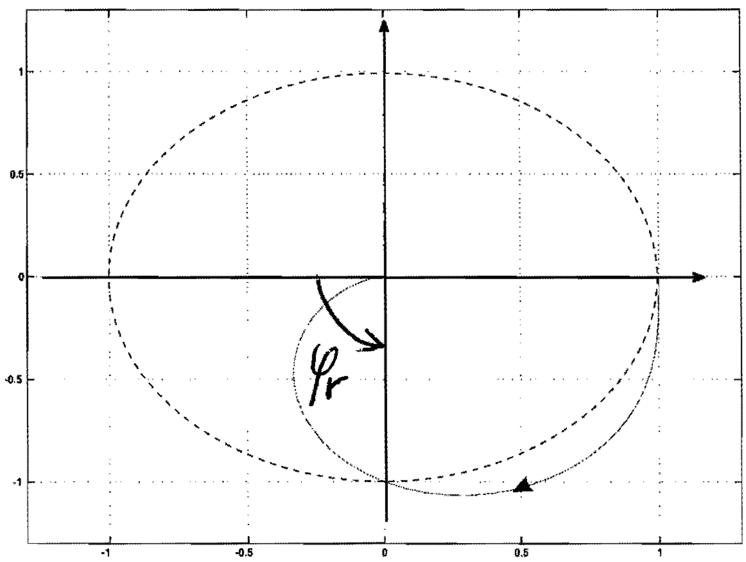


Figure 1.2: Nyquist plot

$$A_r \approx \frac{1}{'0'} \rightarrow \infty \tag{1.1}$$

$$\phi_r = 90^0 \tag{1.2}$$

$A_r > 1$ und $\phi_r > 0 \Rightarrow$ stable

Alternative: The conditions for the use of the specific Nyquist criteria are fulfilled. The closed loop is asymptotically stable because the critical point (0,-1) is located on the left side of the nyquist plot.

1e) (3 Points)
A system with PIT_1-behavior is controlled by a transfer element with D-behavior using negativ feedback (T, T_I, K, T_D). According to the qualitatively drawn root locus of the system, explain if the system can have an asymptotical stable behavior.

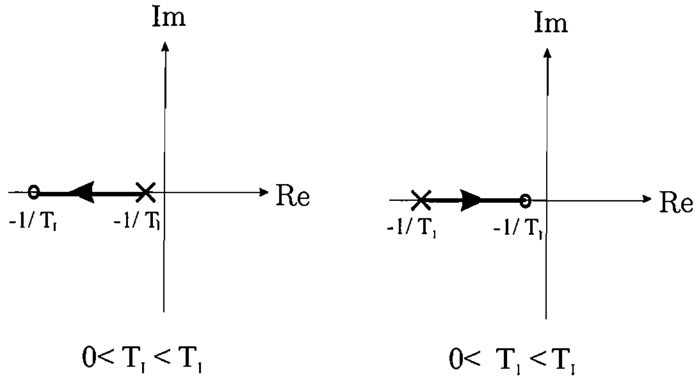


Figure 1.3: Ortskurve

The system is asymptotically stable.

1f) (2 Points)
The root-locus plot of an open loop system $G_0(s)$ with negative feedback is given in Figure 1.4. Due to a mistake, a trainee in the company who is responsible for the design of the controller, miscalculated the root locus, so the negative feedback is taken as positive feedback due to a sign error. Does this affect the stability of the closed-loop system? Explain your answer using a root loci plot.

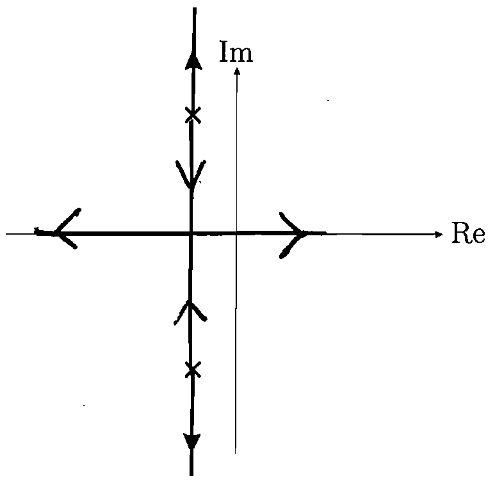


Figure 1.4: Root-locus plot

The system can be stable for specific K. For other K the system behavior is unstable.

$\sum$ □

Problem 2 (20 Points)
The control loop shown in Figure 2.1 consists of a controller $G_{\mathrm{R}}(s)$, a plant $G_{\mathrm{S}}(s)$, and a transfer element $G_{\mathrm{M}}(s)$, which describes the dynamic behavior of a measuring device.

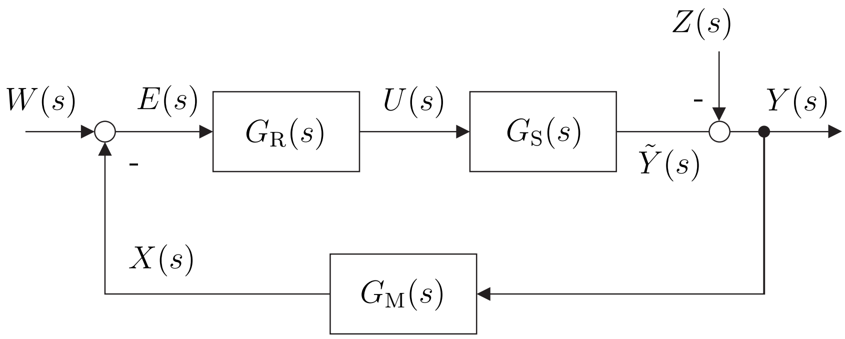


Figure 2.1: Block diagram

The transfer behavior of the plant $G_{\mathrm{S}}(s)$ can be described by the transfer function

$$G_{\mathrm{S}}(s) = \frac{4}{s^2 + 2s - 3}.$$

The transfer behavior of the transfer element $G_{\mathrm{M}}(s)$ can be described by the transfer function

$$G_{\mathrm{M}}(s) = \frac{0.5}{1 + 2s}.$$

2a) (1 Point)
Determine the differential equation of the plant $G_{\mathrm{S}}(s) = \frac{\tilde{Y}(s)}{U(s)}$. Classify the type of transfer behavior (Suppose: $\dot{\tilde{y}}(0) = \tilde{y}(0) = 0$.)?

$$G_{\mathrm{S}}(s) = \frac{\tilde{Y}(s)}{U(s)} = \frac{4}{s^2 + 2s - 3}$$
$$\ddot{\tilde{y}}(t) + 2\dot{\tilde{y}}(t) - 3\tilde{y}(t) = 4u(t)$$

PT2-behavior

2b) (1 Point)
Give the resulting differential equation of the described transfer function $G_{\mathrm{M}}(s)$. Classify the type of transfer behavior (Suppose: $x(0) = 0$.)?

$$G_{\mathrm{M}}(s) = \frac{X(s)}{Y(s)} = \frac{0,5}{2s + 1}$$
$$2\dot{x}(t) + x(t) = 0,5y(t)$$

PT1-behavior

2c) (2 Points)
Determine the disturbance transfer function $G_Z(s) = \frac{Y(s)}{Z(s)}$ with

$$G_R(s) = K_P, \quad G_M(s) = \frac{2}{s+2} \quad \text{and} \quad G_S(s) = \frac{1}{s^2+3s+1}.$$

$$G_Z(s) = -\frac{Y(s)}{Z(s)} = \frac{-1}{1 + G_M G_R G_S}$$
$$G_Z = -\frac{(s^2+3s+1)(s+2)}{s^3+5s^2+7s+2+2K_P}$$

For the subtasks 2d), 2e), and 2f) use the following reference transfer function $G_W(s)$ of a standard control loop

$$G_W(s) = \frac{4K_P(1+s)}{-s^3-2s^2-\tilde{T}s-4+K_P}$$

with $\tilde{T} > 0$ and $K_P > 0$.

2d) (3 Points)
Determine the admissible range of the controller gain K_P for asymptotic stable closed loop system behavior depending on the time constant $\tilde{T}$.

$P(s) = -s^3 - 2s^2 - \tilde{T}s - 4 + K_P$

Necessary cond.: coefficients a_i have identical signs
$a_i > 0 : 4 - K_P > 0 \quad \Longrightarrow \quad K_P < 4$

Necessary cond.: $|H_i| > 0$

$$H = \begin{bmatrix} 2 & 4-K_P & 0 \\ 1 & \tilde{T} & 0 \\ 0 & 2 & 4-K_P \end{bmatrix}$$

$H_1 : D_1 = 2$
$H_2 : D_2 = 2\tilde{T} - 4 + K_P > 0 \quad \Longrightarrow \quad K_P > -2\tilde{T} + 4$
$H_3 : D_3 = (4 - K_P)D_2 > 0 \quad \Longrightarrow \quad K_P > -2\tilde{T} + 4$

The closed loop system is asymptotically stable for:
$0 < \tilde{T} \leq 2 \Longrightarrow -2\tilde{T} + 4 < K_P < 4$ and
$\tilde{T} > 2 \Longrightarrow 0 < K_P < 4$

2e) (2 Points)
Define the theoretical possible numerical intervall of the values of K_P.

In principle: $K \in \mathbb{R}$

$-\infty < K_\text{P} < 4$

2f) (3 Points)
Determine the stationary final value of the step response $h(t \to \infty)$ of the closed loop system and the steady state error by using the controller gain $K_\text{P} = 1$ and the time constant $\tilde{T} = 5$.

$$H(s) = \frac{G(s)}{s} = \frac{4K_\text{P}(1+s)}{(-s^3 - 2s^2 - \tilde{T}s - 4 + K_\text{P})(s)}$$

$$\begin{aligned} \text{Final value} = \lim_{t\to\infty} h(t) = \lim_{s\to\infty} sH(s) \\ = \frac{-4}{3} \end{aligned}$$

$$\text{Steady state error} = 1 - h(t \to \infty) = \frac{7}{3}$$

For the subtasks 2g) and 2h) the modified system

$$G_{\mathrm{R}}(s) = K(s+2) \quad \text{and} \quad G_{\mathrm{S}}(s) = \frac{s}{(s^2-4)(s-5)}$$

is given.

2g) (1 Point)
Is the open loop system $G_{\mathrm{O}}(s)$ stable? State reasons.

Not stable, because at least one pole has a positive real value.

2h) (7 Points)
The root locus plot shall be used to determine the stability of the closed loop system with negative feedback.

1) Determine the breakaway points s_{vi} of the root locus as well as the number of the branches going to infinity and the corresponding angles (Give approximated values.).

2) Sketch the root locus with the help of the given diagram on the next page and denote the directions of the increasing gain.

$G_{\mathrm{O}}(s) = G_{\mathrm{R}}(s)G_{\mathrm{S}}(s)$

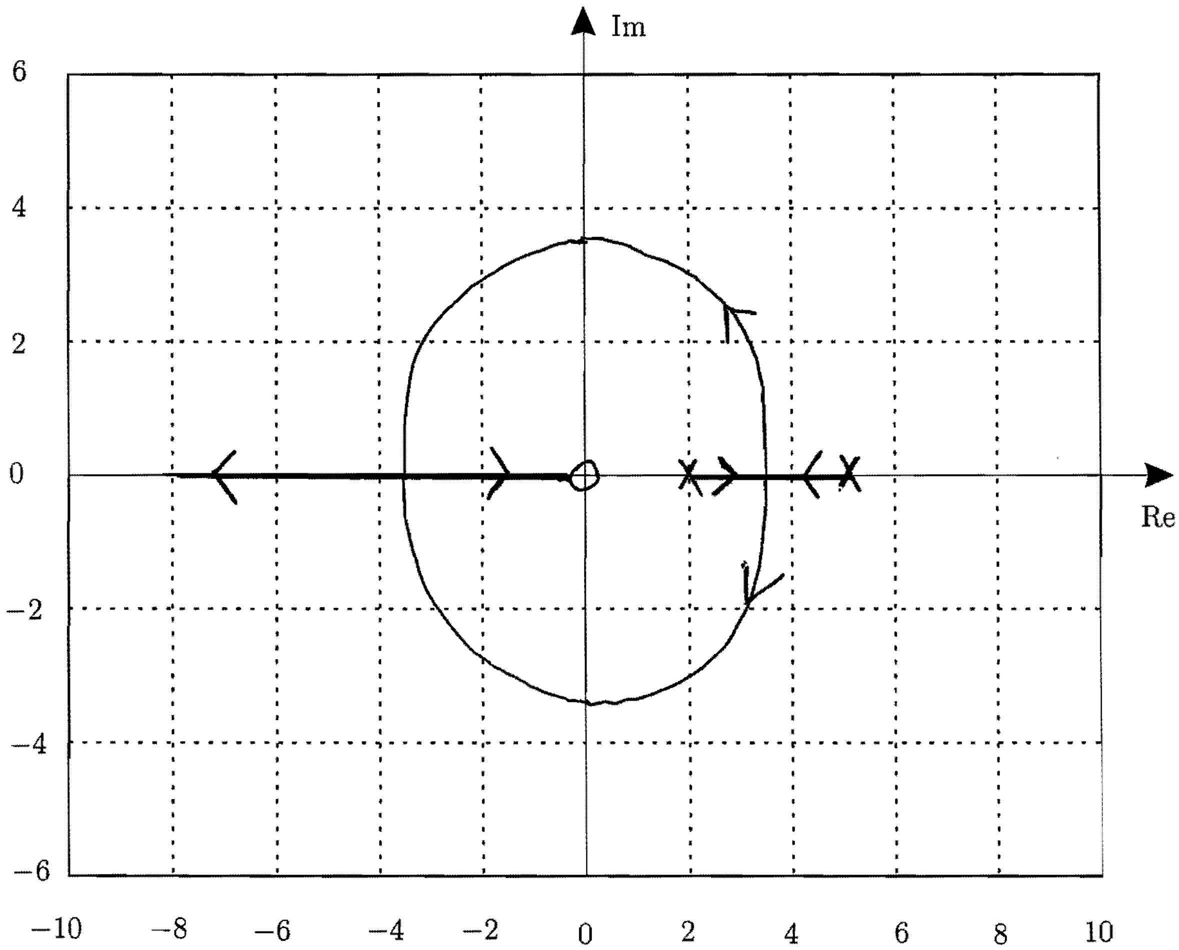
$G_{\mathrm{O}}(s) = \frac{s}{(s-2)(s-5)}$

Breakaway points:

$$\sum_{i=1}^{q} \frac{1}{s_v - n_i} = \sum_{i=1}^{n} \frac{1}{s_v - p_i}$$

$s_v = \pm\sqrt{10}$ ca. $3,16$

Number of the branches going to infinity:
$n - m = 1$
The corresponding angle:
$\varphi = 180°$



$\square$

$\sum \square$

Problem 3 (25 Points)
The dynamic behavior of a human driver regarding her vehicle-guidence behavior can be assumed as $G_{\mathrm{R1}}(s)$

$$G_{\mathrm{R1}}(s) = \frac{V_{\mathrm{M}}(1 + T_{\mathrm{D}}\, s)}{(1 + T_{\mathrm{I}}\, s)}\, e^{s(T_{\mathrm{P}} - \tau_1)}.$$

The longitudinal dynamics of the controlled electric vehicle can be described with G_{S}

$$G_{\mathrm{S}}(s) = \frac{40}{s+2}.$$

3a) (2 points)
Calculate with $V_{\mathrm{M}} = 0.5$, $T_{\mathrm{D}} = 1$, $T_{\mathrm{I}} = \tau_1 = 0.2$, and $T_{\mathrm{P}} = 0$ the phase values of the controller $G_{\mathrm{R1}}(j\omega)$ for $\omega = 0$ and $\omega = +\infty$.

$$G_{\mathrm{R1}}(s) = \frac{0.5(1+s)}{1+0.2s} e^{(-0{,}2s)} \implies G_{\mathrm{R1}}(jw) = \frac{0.5(1+jw)}{1+0.2jw} e^{(-0.2jw)}$$

$i)\ w = 0:$

$$G_{\mathrm{R1}}(j0) = \frac{0.5(1+j0)}{1+0.2j0} e^{(-0.2j0)} = 0.5 \implies arg\ G_{\mathrm{R1}}(j0) = 0^\circ$$

$ii)\ w = +\infty:$

$$G_{\mathrm{R1}}(j\infty) = \frac{0.5(1+j\infty)}{1+0.20j\infty} e^{(-0.20j\infty)} \implies arg\ G_{\mathrm{R1}}(j\infty) = -\infty$$

3b) (5 points)
For the given values of V_{M}, T_{D}, T_{I}, τ_1 as well as T_{P} from subtask 3a), sketch quantitatively the bode diagram of the open loop system $G_{\mathrm{O1}}(s)$ in Figure 3.1 and denote the gradients of the magnitude asymptotes (dB/dec.) and the cut-off frequencies.

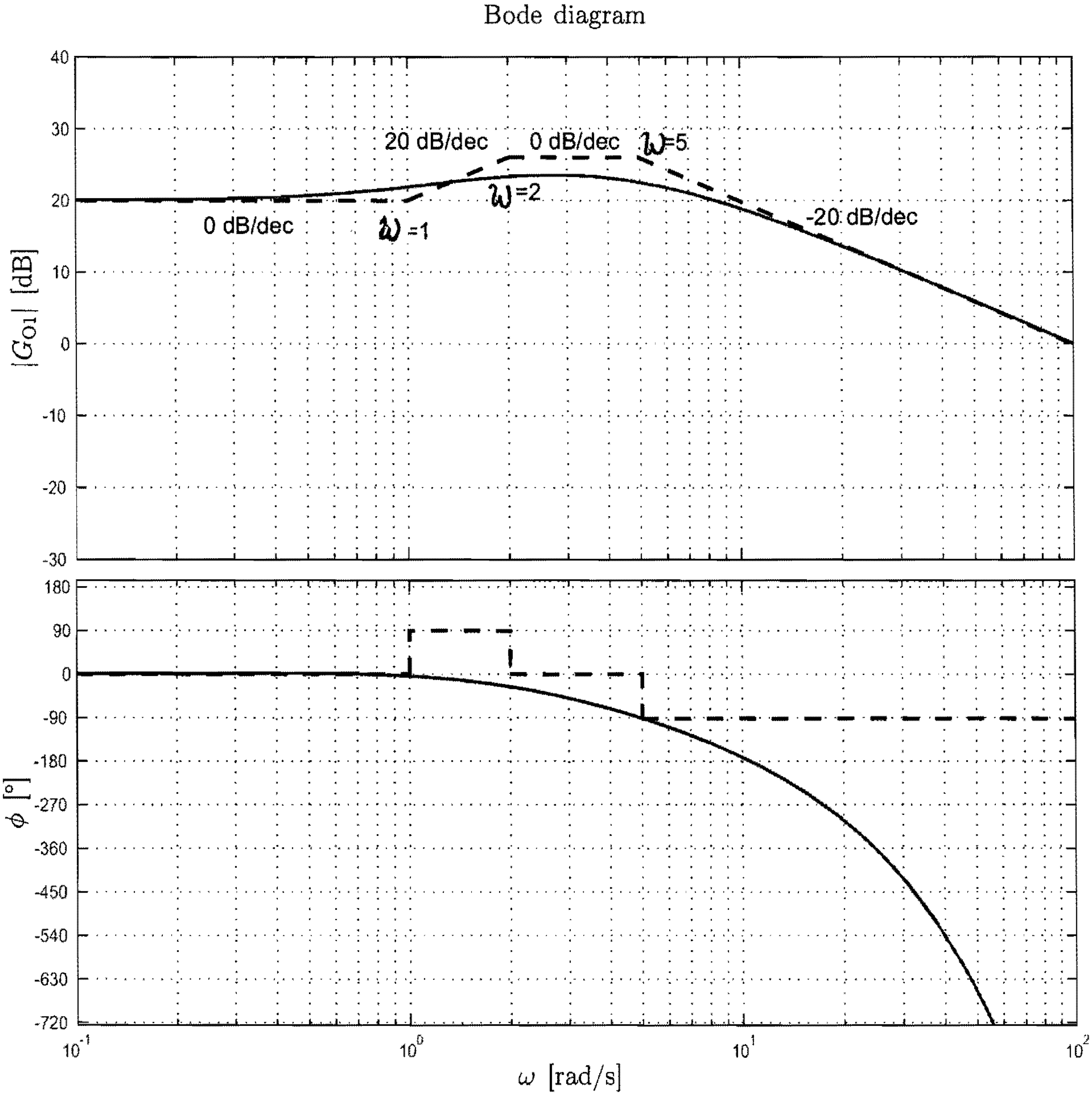


Figure 3.1: Bode diagram of subtask 3b)

3c) (2 points)
Is the control loop $G_{\mathrm{O1}}(s)$ determined in subtask 3b) without time delay a phase-minimum system? Does the time delay change this classification? State reasons.

Yes, because the zeros and poles are stable.

Yes, the time delay leads to a non-minimum-phase system [real part (pole, zeros)<0].

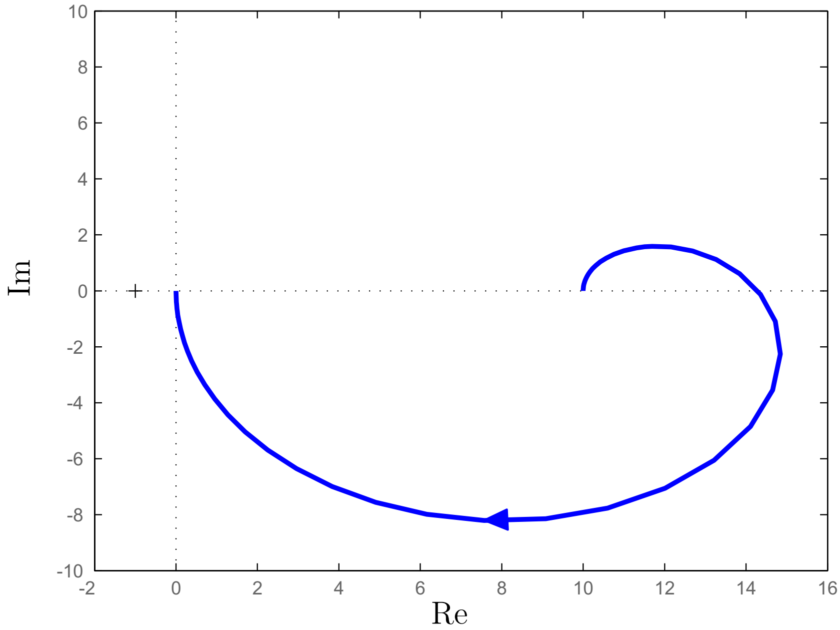
3d) (5 points)
Neclecting the delay time leads to the following transfer function of the driver

$$G_{\mathrm{R2}}(s) = \frac{0.5(1+s)}{(1+0.2\,s)}.$$

Determine by means of nyquist plot of the corresponding system, whether the driver described by G_{R2} can stabilize the vehicle G_{S} with negative feedback. If not, determine the number of unstable poles in the closed loop system.

$$G_{\mathrm{o2}}(s) = G_{\mathrm{R2}}(s)G_{\mathrm{S}}(s) = \frac{0.5(1+s)}{(1+0.2\,s)}\frac{40}{s+2)} = \frac{10(s+1)}{(1+0.2\,s)(1+0.5\,s)}.$$

Nyquist plot



According to the "left-hand-rule" the closed loop system is stable, because the nyquist plot does not encircle the point $-1+j0$.

3e) (3 points)

By drinking alcohol the behavior of the driver is influenced. For the modeling, an additional time delay is considered. The behavior of the driver (controller) is now described by

$$G_{\mathrm{R3}}(s) = \frac{V_{\mathrm{M}}(1 + T_{\mathrm{D}}\, s)}{(1 + T_{\mathrm{I}}\, s)}\, e^{s(T_{\mathrm{P}} - \tau_2)}$$

with $V_{\mathrm{M}} = 0.5$, $T_{\mathrm{D}} = 0$, $T_{\mathrm{I}} = 0.2$, $\tau_2 = 1.2$, and $T_{\mathrm{P}} = 0$.

Determine according to the given bode diagram of the open loop $G_{\mathrm{O3}}(s)$ shown in Figure 3.2 the stability of the closed loop system with negative feedback. State reasons.

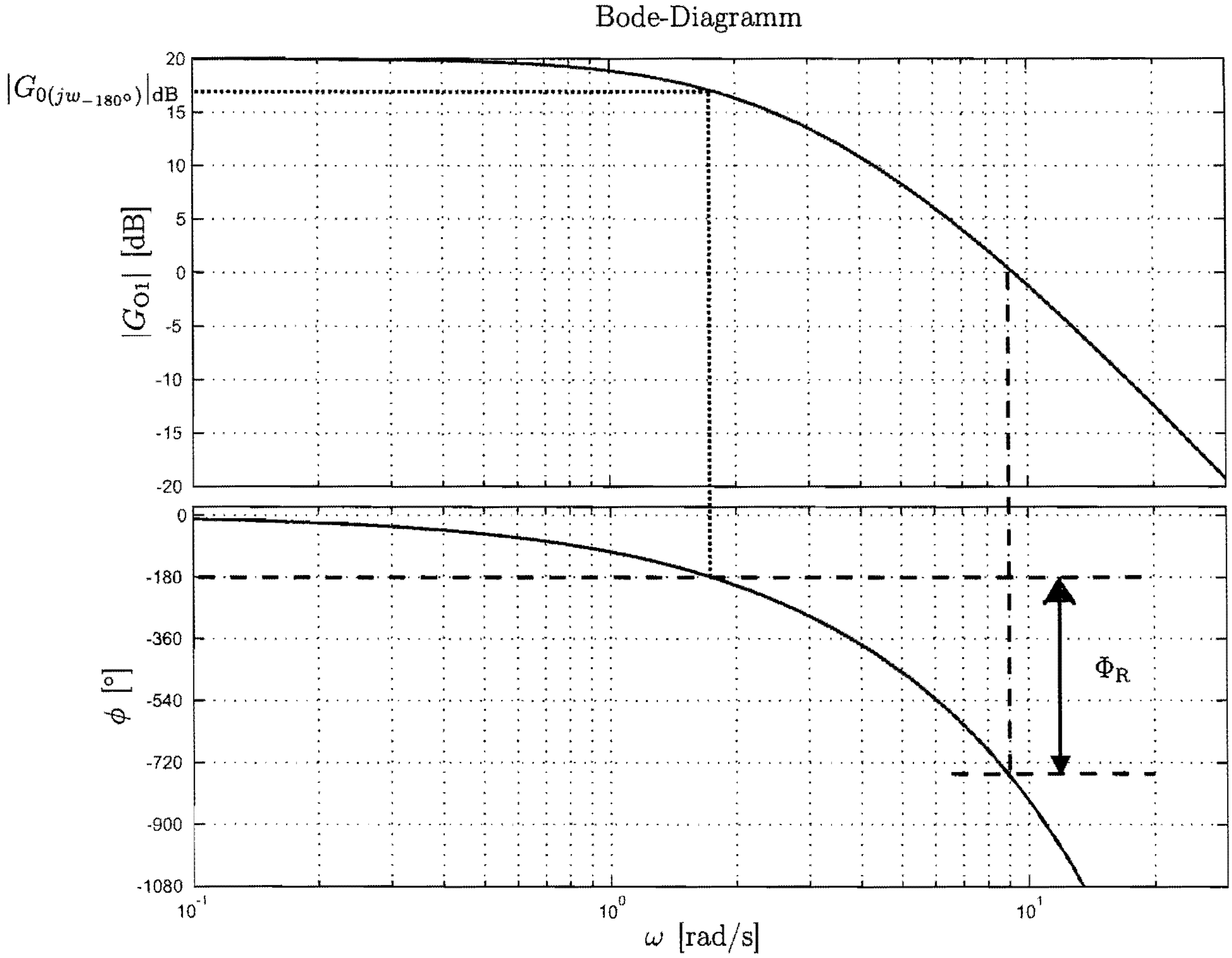


Figure 3.2: Bode-Diagramm zur Teilaufgabe 3e)

Phase margin $\Phi_\mathrm{R} < 0°$

$\Longrightarrow$ The closed loop system is unstable.

alternatively:

$|G_{0(jw_{-180°})}|_\mathrm{dB} \doteq 17\mathrm{dB} \implies |G_{0(jw_{-180°})}| > 1$

The gain margin $A_\mathrm{R} = \dfrac{1}{|G_{0(jw_{-180°})}|} < 1$

$\Longrightarrow$ The closed loop system is unstable.

3f) (8 points)
The bode diagram of a technical system is illustrated in Figure 3.3. Determine the number of zeros and the number of poles. Draw qualitatively the positions of the zeros (with "∘") and poles (with "×") in Figure 3.4. Assign the frequence from the Figure 3.3 qualitatively to the positions of the zeros/poles in Figure 3.4.

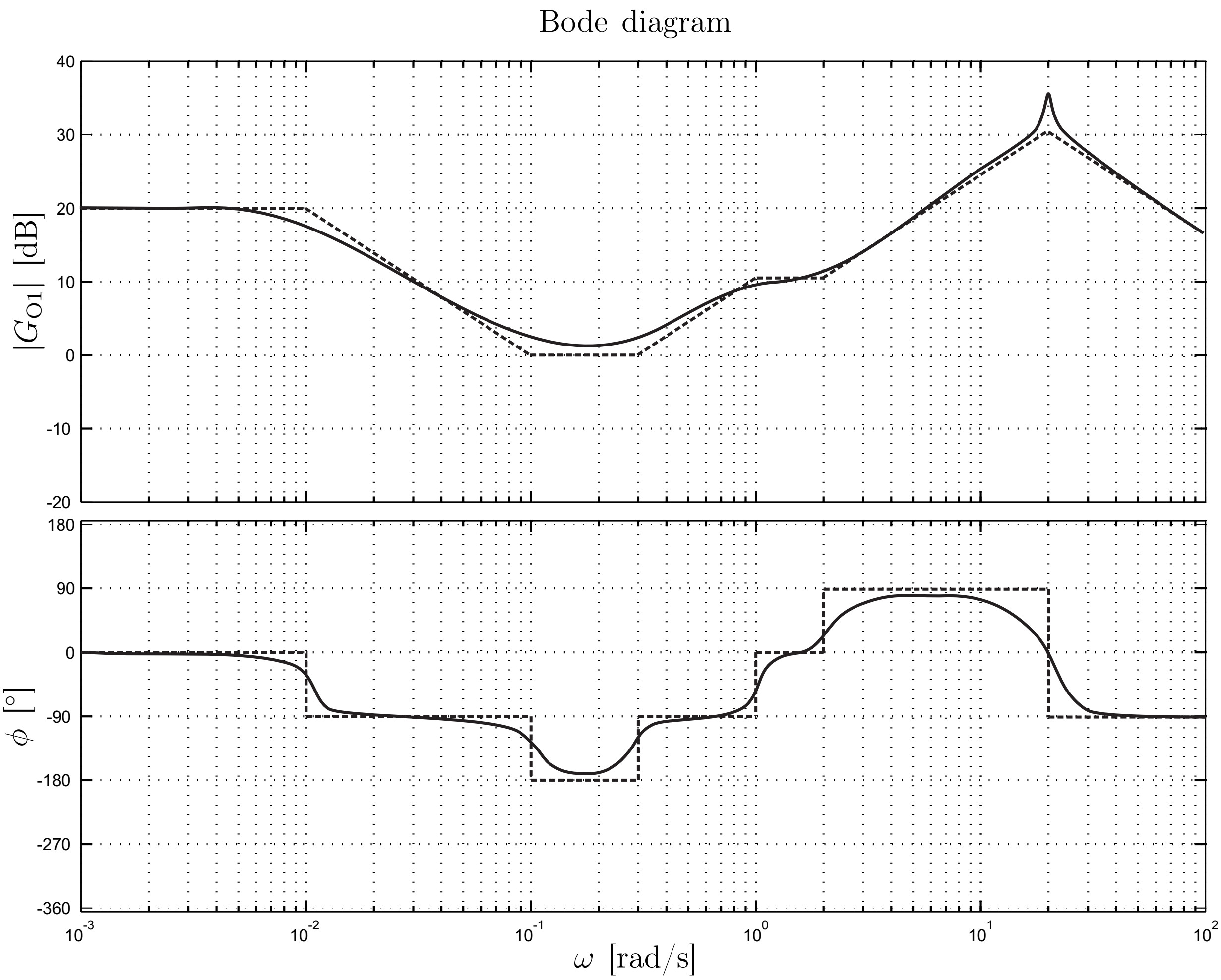


Figure 3.3: Bode diagram of subtask 3f)

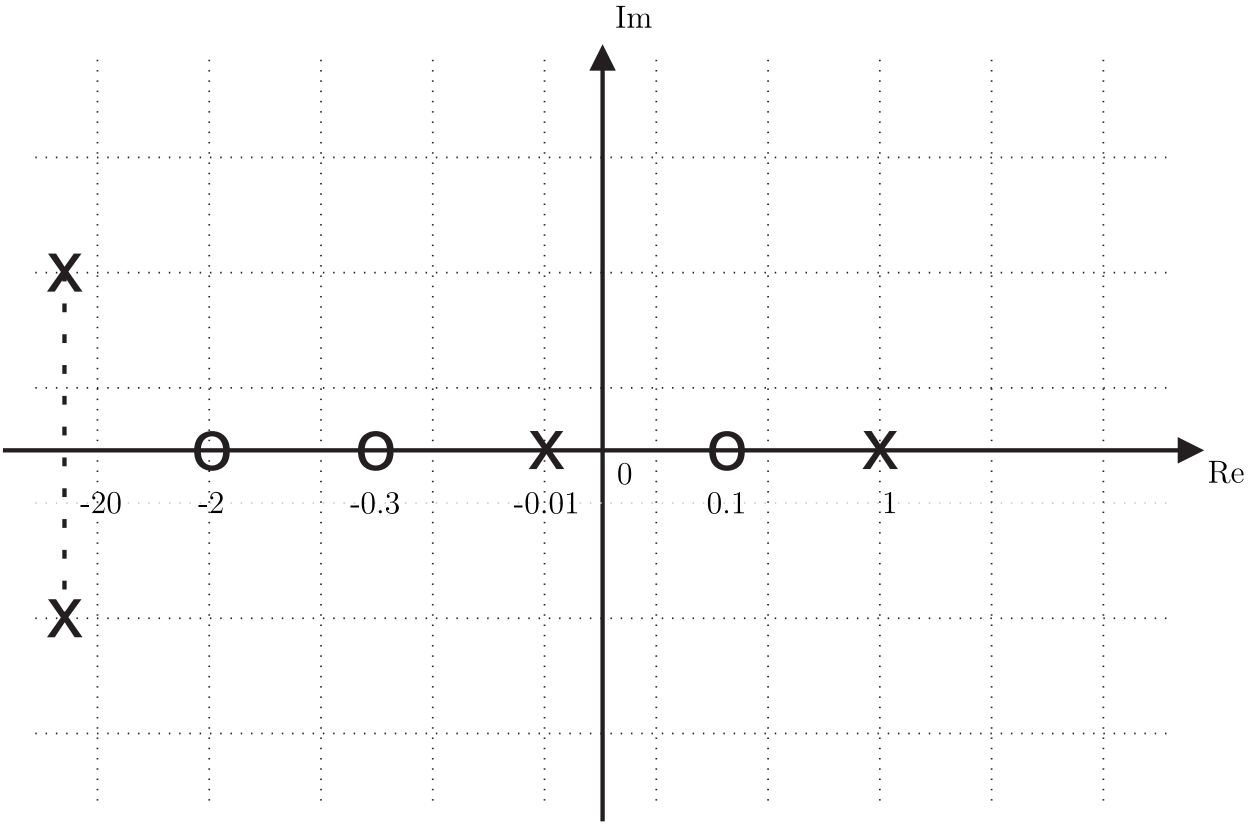


Figure 3.4: Zero/pole positions of subtask 3f)

Number of zeros: 3
Number of poles: 4

$\sum \square$