URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

Efficient Analysis of Complex Modes in Cylindrical Photonic Crystal

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Cylindrical photonic crystal is a topic of active research because of its potential importance in fiber-optic communications [1], nonlinear devices [2]. The modal properties of cylindrical photonic crystal have been extensively investigated using the finite element method combined with PML [3], the finite difference frequency domain method, the multipole method [4]. These numerical methods could be versatilely applied to various microstructured configurations but they are computationally intensive.

In this paper, we shall present a novel full-wave rigorous approach for the vector fields in cylindrical photonic crystals, which consists of layered cylindrical arrays of circular rods symmetrically distributed on each of concentric (eccentric) circular cylindrical surfaces. The method is computationally fast and easy to implement for a wide class of cylindrical photonic crystals. The proposed approach introduces a cylindrical layer model to the array, extracts the reflection and transmission matrices of a cylindrical periodic layer, and then obtains the characteristics of the whole layered structure by using a recursive algorithm [5, 6]. In our formalism we take into account all cylindrical Floquet modes and their interactions through the scattering by each cylindrical layer. In the case of the modal analysis of the guided waves without any initial excitation, we could assume a unique symmetric property of the mode field distribution inherent to the periodicity of the circular rods. In the work we calculate the complex propagation constant for hexagonal arrays and the results will be presented at the symposium.

References

[1] P. St. J. Russell, “Photonic-crystal fibers”, *J. Lightwave Technol.* **24**, 12, 2006, 4729-4749.

[2] J. W"atzel and J. Berakdar, “Topological light fields for highly on-linear charge quantum dynamics and high harmonic generation”, *Optics Express,* **28**, 12, 2020, 19469-19481.

[3] K. Saitoh and M. Koshiba, “Full-vectorial imaginary-distance beam propagation method based on a finite element scheme: application to photonic crystal fibers”, *IEEE Journal of Quantum Electronics,* **38**, 7, 2002, 927-933.

[4] T. P. White et.al, “Multipole method for microstructured optical fibers. I. Formulation”, *J. Opt. Soc. Am. B,* **19**, 10, 2002, 2322-2330.

[5] V. Jandieri, K. Yasumoto and Y. Liu, “Directivity of radiation of a dipole source coupled to cylindrical electromagnetic bandgap structures,” *J. Opt. Soc. Am. B,* **29**, 9, 2012, 2622-2629.

[6] V. Jandieri, P. Baccarelli, G. Valerio and G. Schettini, “1-D Periodic lattice sums for complex and leaky waves in 2-D structures using higher-order Ewald formulation,“ *IEEE Transactions on Antennas and Propagation,* **67**, 4, 2019, 2364 - 2378.

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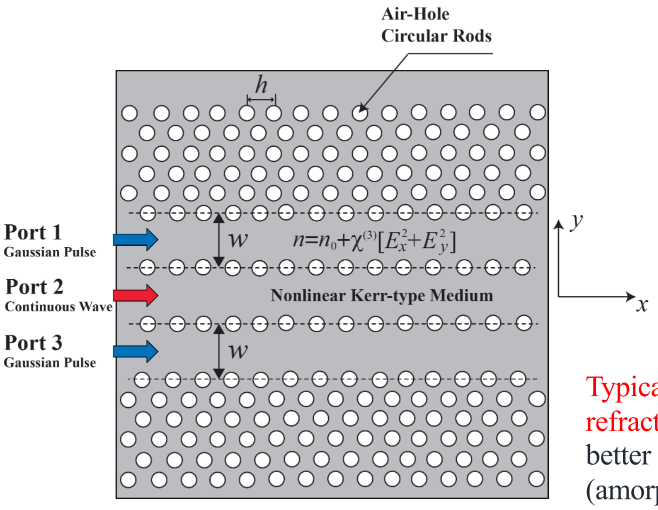
Introduction

- Motivation.
- Novelty of the Work.
- Formulation of the Problem.
- Numerical Results and Discussions.

Motivation

- All-optical logic gates, which are responsible for various logical operations in all-optical circuits, play a key role in ultrafast optical signal processing.

- We demonstrated the realization of true all-optical NOT, AND and NAND logic gates using **gap-solitons** in photonic crystal waveguides composed of an experimentally feasible **planar** air-hole type hexagonal structure.



n_0 = 2.95 (linear refractive index)
r = 0.32h (radius of the rods)
w = 1.73h (width of the waveguide)

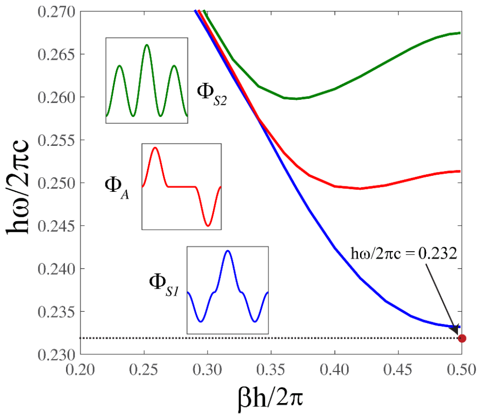
Bandgap for H-modes (H_z, E_x, E_y). Air-Hole type PhCs. Length of the device is 30h.

Typical semiconductor material (*Si*, *GaAs*, *InP*) with relative high refractive index is studied. For practical application would be better to use the materials with enhanced third order nonlinearities (amorphous *Si*, hybrid organic-*Si*-on-insulator compounds).

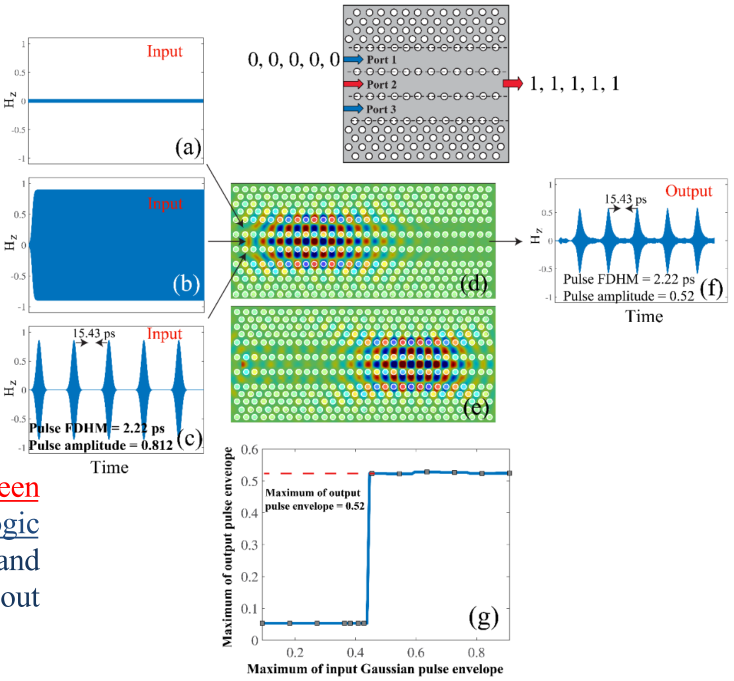
V. Jandieri, R. Khomeriki, T. Onoprishvili, D.H. Werner, J. Berakdar and D. Erni, "Functional All-Optical Logic Gates for True Time-domain Signal Processing in Nonlinear Photonic Crystal Waveguides," ***Optics Express,*** **vol. 28, no. 12, pp. 18317-18331, 2020.**

Motivation

A key element in the working concept of the proposed all-optical logic gates is the virtually "***perfect digitalization***" of the involved time-domain signals. All investigated gate topologies operate with temporal bandgap solitons having stable pulse envelopes during signal processing, which is one of main advantages of the proposed working concept of the device.



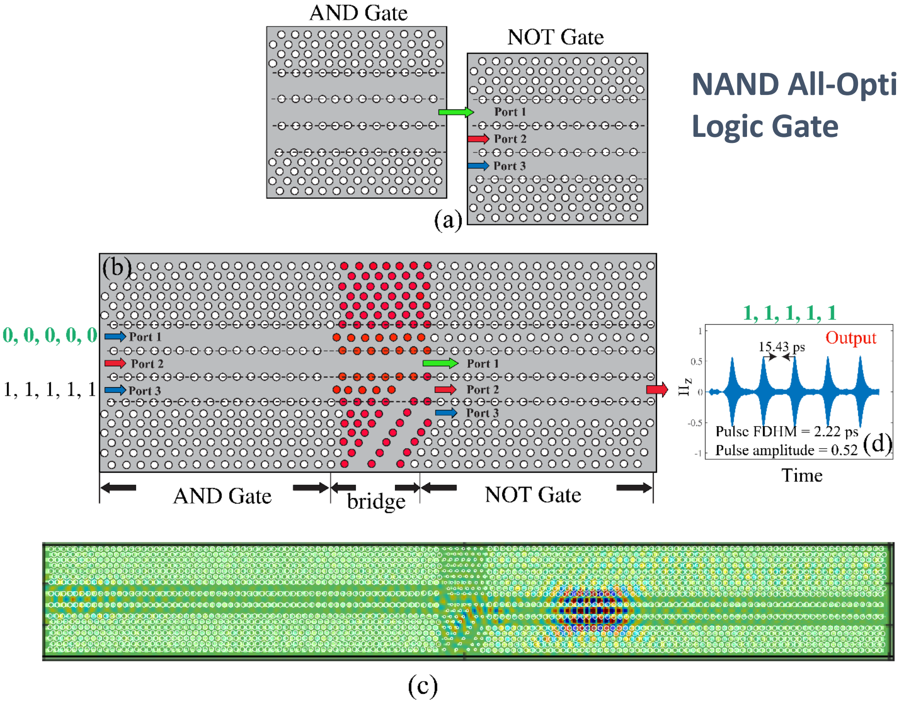
Realization of fully optical NOT logic gate



Similar characteristics between I/O has been observed (key element for true all-optical logic gates). Relative error between the output and input signals (its amplitude and width) is about 7 % - 9 %.

Motivation

In the proposed setup, there is no need to amplify the output signal after each logic operation, and can be directly use it as a new input signal for another logical operation.



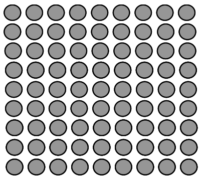
NAND All-Optical Logic Gate

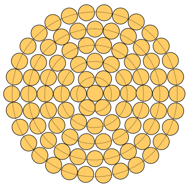
Can we realize logical operations in Photonic Crystals with cylindrical symmetry?

V. Jandieri, R. Khomeriki, T. Onoprishvili, D.H. Werner, J. Berakdar and D. Erni, "Functional All-Optical Logic Gates for True Time-domain Signal Processing in Nonlinear Photonic Crystal Waveguides," ***Optics Express,*** **vol. 28, no. 12, pp. 18317-18331, 2020.**

Novelty of the Work

Cylindrical Photonic Crystals, which are formed by circular rods periodically distributed on layered concentric or eccentric circular [1].





Photonic Crystal Fibers;
Microstructured Optical Fibers;

Directive antennas;
Beam-switching Antennas;
Cylindrical EBG-based Antennas.

1. For isotropic scatterer the reflection matrix of single planar layer always satisfies the following equality: $\mathbf{R}_{\nu,\nu-1} = \mathbf{R}_{\nu-1,\nu}$ (Reciprocity Relation).
2. Each Floquet mode is excited only in some particular frequency range.

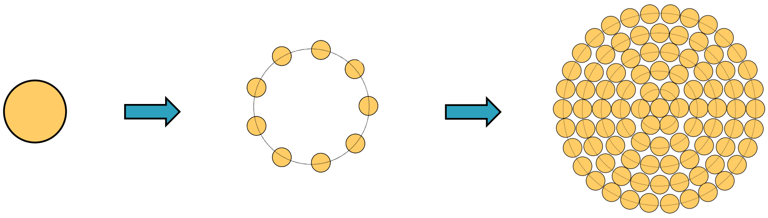
1. Since the reflection matrices are expressed in terms of the cylindrical waves, even for isotropic scatterer: $\mathbf{R}_{\nu,\nu-1} \neq \mathbf{R}_{\nu-1,\nu}$.
2. All orders of cylindrical harmonic waves are excited. However, due to the resonances and stopbands nature some of them are enhanced, whereas others are strongly suppressed.

[1] Vakhtang Jandieri and Kiyotoshi Yasumoto, "Scattering and Guidance by Layered Cylindrically Periodic Arrays of Circular Cylinders" ***Advances in Mathematical Methods for Electromagnetics*****, published by the Institution of Engineering and Technology (IET), Editors: Kazuya Kobayashi and Paul Denis Smith, pp. 515-545, 2021.**

Formulation of the Problem

- We have developed a semi-analytical method, which can be applied to the **guiding, scattering and radiation problems** in cylindrical periodic (or EBG) structures.

- The approach uses:
  - **Transition Matrix (T-matrix) of a circular rod;**
  - **Translation matrices for cylindrical waves;**
  - **Reflection and transmission matrices based on cylindrical waves for each layer;**
  - **Generalized reflection and transmission matrices for multi-layered structure.**



[1] Vakhtang Jandieri and Kiyotoshi Yasumoto, "Scattering and Guidance by Layered Cylindrically Periodic Arrays of Circular Cylinders" *Advances in Mathematical Methods for Electromagnetics*, published by the Institution of Engineering and Technology (IET), Editors: Kazuya Kobayashi and Paul Denis Smith, pp. 515-545, 2021.

Formulation of the Problem

In the (ν)-th region:

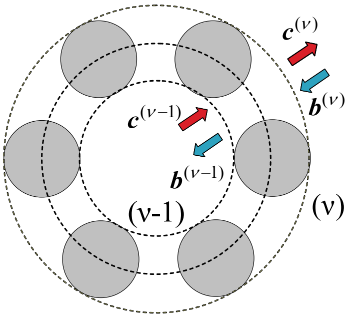
$$\left.\begin{aligned}\tilde{E}_z^{out(\nu)}(\rho,\varphi,\xi) &= \boldsymbol{\Psi}^{\mathrm{T}}\cdot\boldsymbol{c}^{e(\nu)}\\ \tilde{H}_z^{out(\nu)}(\rho,\varphi,\xi) &= \boldsymbol{\Psi}^{\mathrm{T}}\cdot\boldsymbol{c}^{h(\nu)}\end{aligned}\right\}$$ *Outgoing* waves

$$\left.\begin{aligned}\tilde{E}_z^{inc(\nu)}(\rho,\varphi,\xi) &= \boldsymbol{\Phi}^{\mathrm{T}}\cdot\boldsymbol{b}^{e(\nu)}\\ \tilde{H}_z^{inc(\nu)}(\rho,\varphi,\xi) &= \boldsymbol{\Phi}^{\mathrm{T}}\cdot\boldsymbol{b}^{h(\nu)}\end{aligned}\right\}$$ *Incoming* waves

$$\boldsymbol{\Psi} = [H_m^{(1)}(\kappa\rho)\,\mathrm{e}^{im\varphi}], \quad \boldsymbol{\Phi} = [J_m(\kappa\rho)\,\mathrm{e}^{im\varphi}]$$

$$\boldsymbol{c}^{i(\nu)} = [c_m^{i(\nu)}], \quad \boldsymbol{b}^{i(\nu)} = [b_m^{i(\nu)}] \qquad (i = e, h)$$

$$\kappa = \sqrt{k^2 - \xi^2}, \quad \xi: \text{ spectral parameter}$$



In the $(\nu-1)$-th region:

$$\tilde{E}_z^{out(\nu-1)}(\rho,\varphi,\xi) = \boldsymbol{\Psi}^{\mathrm{T}}\cdot\boldsymbol{c}^{e(\nu-1)}, \quad \tilde{H}_z^{out(\nu-1)}(\rho,\varphi,\xi) = \boldsymbol{\Psi}^{\mathrm{T}}\cdot\boldsymbol{c}^{h(\nu-1)}$$

$$\tilde{E}_z^{inc(\nu-1)}(\rho,\varphi,\xi) = \boldsymbol{\Phi}^{\mathrm{T}}\cdot\boldsymbol{b}^{e(\nu-1)}, \quad \tilde{H}_z^{inc(\nu-1)}(\rho,\varphi,\xi) = \boldsymbol{\Phi}^{\mathrm{T}}\cdot\boldsymbol{b}^{h(\nu-1)}$$

$$\boldsymbol{c}^{i(\nu-1)} = [c_m^{i(\nu-1)}], \quad \boldsymbol{b}^{i(\nu-1)} = [b_m^{i(\nu-1)}] \qquad (i = e, h)$$

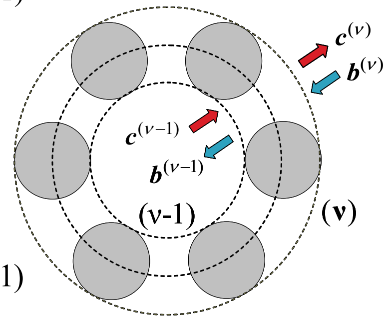
Formulation of the Problem

Reflection and transmission matrices

For the incidence of outgoing wave with $\overline{\boldsymbol{c}}^{(\nu-1)}$

$$\overline{\boldsymbol{b}}^{(\nu-1)} = \begin{bmatrix} \boldsymbol{b}^{e(\nu-1)} \\ \boldsymbol{b}^{h(\nu-1)} \end{bmatrix} = \mathbf{R}_{\nu-1,\nu} \cdot \begin{bmatrix} \boldsymbol{c}^{e\nu-1)} \\ \boldsymbol{c}^{h(\nu-1)} \end{bmatrix} = \mathbf{R}_{\nu-1,\nu} \cdot \overline{\boldsymbol{c}}^{(\nu-1)}$$

$$\overline{\boldsymbol{c}}^{(\nu)} = \begin{bmatrix} \boldsymbol{c}^{e(\nu)} \\ \boldsymbol{c}^{h(\nu)} \end{bmatrix} = \mathbf{F}_{\nu,\nu-1} \cdot \begin{bmatrix} \boldsymbol{c}^{e(\nu-1)} \\ \boldsymbol{c}^{h(\nu-1)} \end{bmatrix} = \mathbf{F}_{\nu,\nu-1} \cdot \overline{\boldsymbol{c}}^{(\nu-1)}$$



For the incidence of incoming wave with $\overline{\boldsymbol{b}}^{(\nu)}$

$$\overline{\boldsymbol{c}}^{(\nu)} = \begin{bmatrix} \boldsymbol{c}^{e(\nu)} \\ \boldsymbol{c}^{h(\nu)} \end{bmatrix} = \mathbf{R}_{\nu,\nu-1} \cdot \begin{bmatrix} \boldsymbol{b}^{e(\nu)} \\ \boldsymbol{b}^{h(\nu)} \end{bmatrix} = \mathbf{R}_{\nu,\nu-1} \cdot \overline{\boldsymbol{b}}^{(\nu-1)}$$

$$\overline{\boldsymbol{b}}^{(\nu-1)} = \begin{bmatrix} \boldsymbol{b}^{e(\nu-1)} \\ \boldsymbol{b}^{h(\nu-1)} \end{bmatrix} = \mathbf{F}_{\nu-1,\nu} \cdot \begin{bmatrix} \boldsymbol{b}^{e(\nu)} \\ \boldsymbol{b}^{h(\nu)} \end{bmatrix} = \mathbf{F}_{\nu-1,\nu} \cdot \overline{\boldsymbol{b}}^{(\nu)}$$

Formulation of the Problem

For dielectric circular rods :

hybrid modes [1].

$$\mathbf{R}_{\nu-1,\nu} = \begin{bmatrix} \mathbf{R}^{ee}_{\nu-1,\nu} & \mathbf{R}^{eh}_{\nu-1,\nu} \\ \mathbf{R}^{he}_{\nu-1,\nu} & \mathbf{R}^{hh}_{\nu-1,\nu} \end{bmatrix}, \quad \mathbf{R}_{\nu,\nu-1} = \begin{bmatrix} \mathbf{R}^{ee}_{\nu,\nu-1} & \mathbf{R}^{eh}_{\nu,\nu-1} \\ \mathbf{R}^{he}_{\nu,\nu-1} & \mathbf{R}^{hh}_{\nu,\nu-1} \end{bmatrix}$$

$$\mathbf{F}_{\nu,\nu-1} = \begin{bmatrix} \mathbf{F}^{ee}_{\nu,\nu-1} & \mathbf{F}^{eh}_{\nu,\nu-1} \\ \mathbf{F}^{he}_{\nu,\nu-1} & \mathbf{F}^{hh}_{\nu,\nu-1} \end{bmatrix}, \quad \mathbf{F}_{\nu-1,\nu} = \begin{bmatrix} \mathbf{F}^{ee}_{\nu-1,\nu} & \mathbf{F}^{eh}_{\nu-1,\nu} \\ \mathbf{F}^{he}_{\nu-1,\nu} & \mathbf{F}^{hh}_{\nu-1,\nu} \end{bmatrix}$$

For perfect conductor circular rods :

modes are decoupled [1].

$$\mathbf{R}_{\nu-1,\nu} = \begin{bmatrix} \mathbf{R}^{ee}_{\nu-1,\nu} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}^{hh}_{\nu-1,\nu} \end{bmatrix}, \quad \mathbf{R}_{\nu,\nu-1} = \begin{bmatrix} \mathbf{R}^{ee}_{\nu,\nu-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}^{hh}_{\nu,\nu-1} \end{bmatrix}$$

$$\mathbf{F}_{\nu,\nu-1} = \begin{bmatrix} \mathbf{F}^{ee}_{\nu,\nu-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^{hh}_{\nu,\nu-1} \end{bmatrix}, \quad \mathbf{F}_{\nu-1,\nu} = \begin{bmatrix} \mathbf{F}^{ee}_{\nu-1,\nu} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^{hh}_{\nu-1,\nu} \end{bmatrix}$$

[1] Vakhtang Jandieri and Kiyotoshi Yasumoto, “Scattering and Guidance by Layered Cylindrically Periodic Arrays of Circular Cylinders” *Advances in Mathematical Methods for Electromagnetics*, published by the Institution of Engineering and Technology (IET), Editors: Kazuya Kobayashi and Paul Denis Smith, pp. 515-545, 2021.

Formulation of the Problem

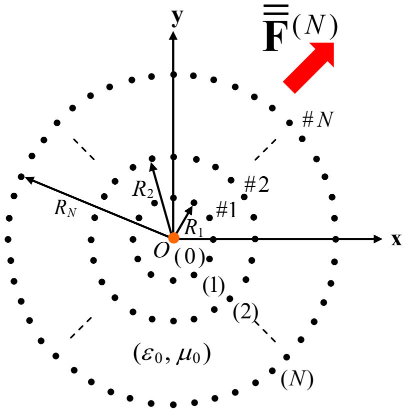
Generalized Reflection and Transmission Matrices

$$\overline{\overline{\mathbf{R}}}_{\nu-1,\nu} = \mathbf{R}_{\nu-1,\nu} + \mathbf{F}_{\nu-1,\nu}\overline{\overline{\mathbf{R}}}_{\nu,\nu+1}\overline{\overline{\boldsymbol{\Gamma}}}_{\nu,\nu-1}$$

$$\overline{\overline{\boldsymbol{\Gamma}}}_{\nu,\nu-1} = (\mathbf{I} - \mathbf{R}_{\nu,\nu-1}\overline{\overline{\mathbf{R}}}_{\nu,\nu-1})^{-1}\mathbf{F}_{\nu,\nu-1}$$

$$\overline{\overline{\mathbf{R}}}_{N,N+1} = \mathbf{0}$$

For antenna problem



$\overline{\overline{\mathbf{R}}}_{\nu-1,\nu}$: Generalized reflection matrix viewed from region $(\nu-1)$ to the whole outer regions from (ν) to (N)

$$\boxed{\overline{\overline{\mathbf{F}}}^{(N)} = \overline{\overline{\boldsymbol{\Gamma}}}_{N,N-1}\overline{\overline{\boldsymbol{\Gamma}}}_{N-1,N-2}\cdots\overline{\overline{\boldsymbol{\Gamma}}}_{2,1}\overline{\overline{\boldsymbol{\Gamma}}}_{1,0}}$$

Formulation of the Problem

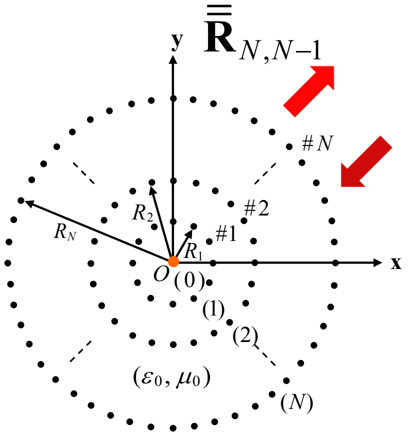
Generalized Reflection and Transmission Matrices

$$\overline{\overline{\mathbf{R}}}_{\nu,\nu-1} = \mathbf{R}_{\nu,\nu-1} + \mathbf{F}_{\nu,\nu-1}\overline{\overline{\boldsymbol{\Gamma}}}_{\nu-1,\nu-2}\overline{\overline{\mathbf{R}}}_{\nu-1,\nu-2}\mathbf{F}_{\nu-1,\nu}$$

$$\overline{\overline{\boldsymbol{\Gamma}}}_{\nu-1,\nu-2} = (\mathbf{I} - \overline{\overline{\mathbf{R}}}_{\nu-1,\nu-2}\mathbf{R}_{\nu-1,\nu})^{-1}$$

$$\overline{\overline{\mathbf{R}}}_{2,1} = \mathbf{R}_{2,1}$$

For guiding problem

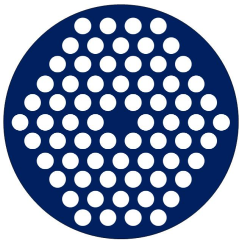


$\overline{\overline{\mathbf{R}}}_{\nu-1,\nu-2}$: Generalized reflection matrix viewed from region $(\nu-1)$ to the whole inner regions from $(\nu-1)$ to (0)

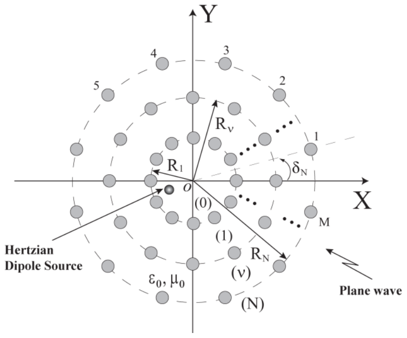
Formulation of the Problem

Schematics for guidance, scattering and radiation problems.

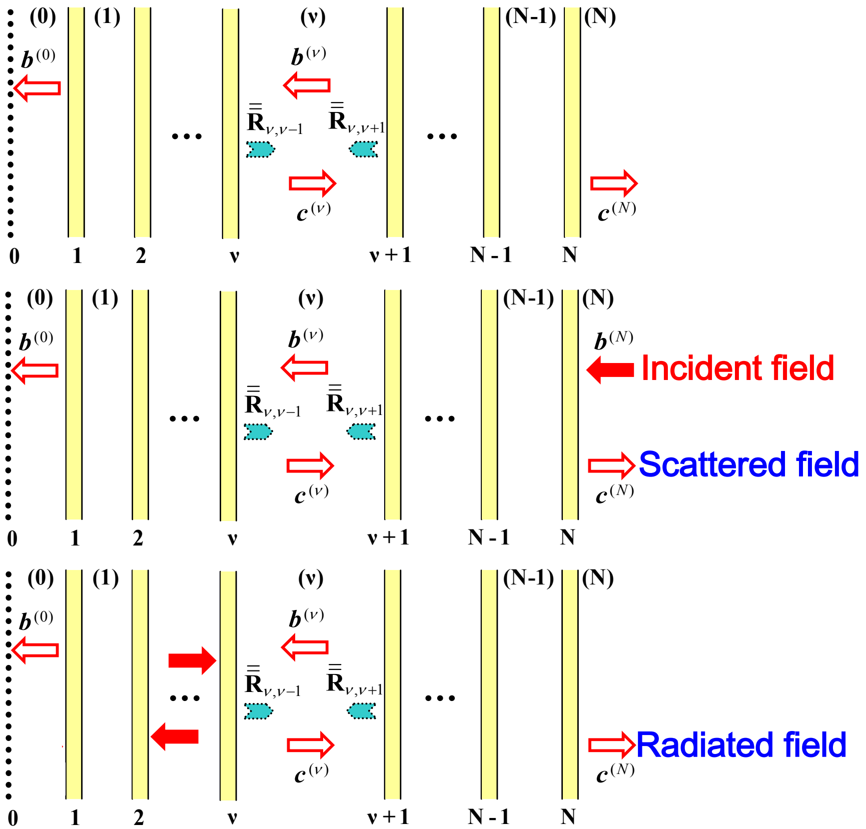
Guiding problems



Scattering problems



Radiation problems



Incident field

Scattered field

Radiated field

Source field

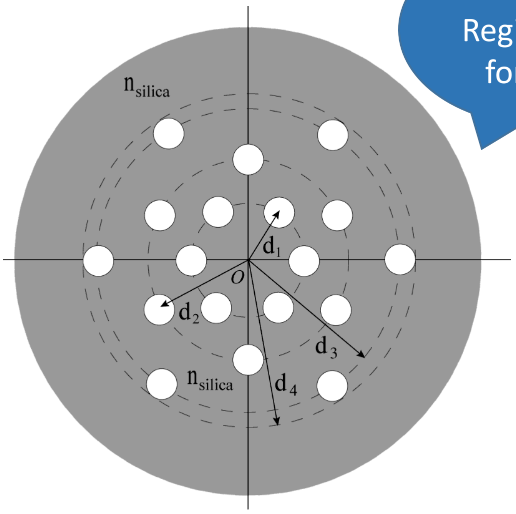
Numerical Results and Discussions

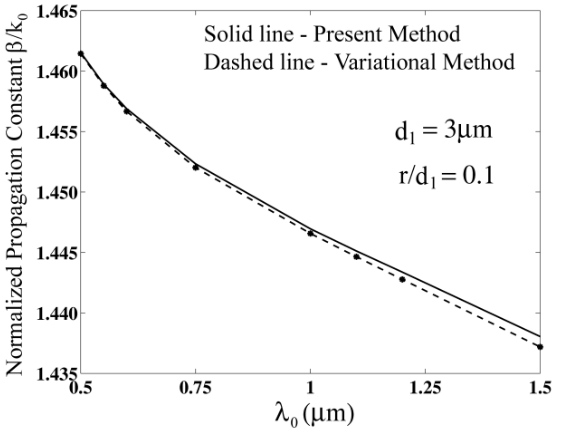
Microstructure Optical Fiber

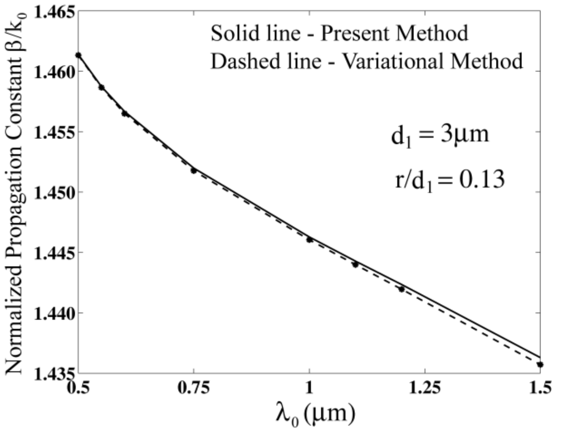
Background medium is pure Silica.
Air-holes are drilled in Silica.

Effective refractive index of cladding is lower than that of core. ***TIR effect.***

No Bandgap Region was formed.

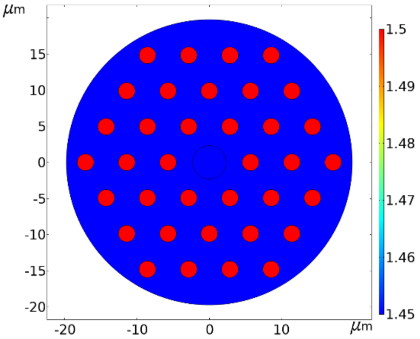




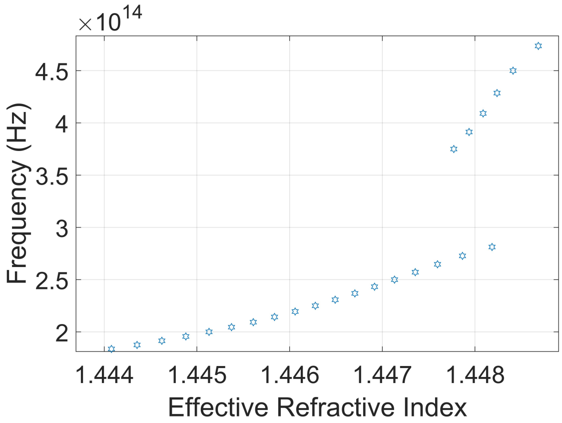


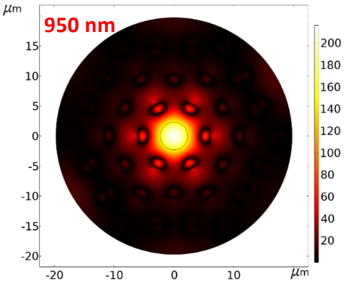
Numerical Results and Discussions

Effective refractive index of cladding is higher than that of core. ***No TIR effect.***

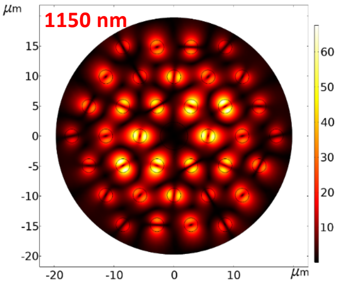


$n_{hole} = 1.485$
$n_{cladding} = 1.449$
$d_{hole} = 2.3$ (micron)
$r_{hole} = 5.7$ (micron)

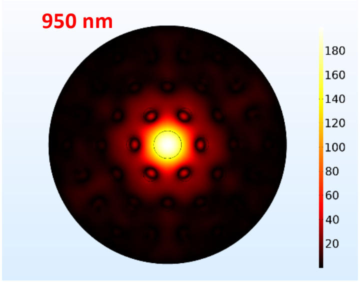




Frequency domain



Frequency domain



Time domain
(after 150 micron)

Thank You!

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Alexander von Humboldt Foundation

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