Received: 22 June 2018 | Accepted: 04 October 2018

DOI: 10.1002/pamm.201800413

Anisotropic wave dispersion and band-gaps in mechanical metamaterials via the relaxed micromorphic model

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We propose a continuum model (the relaxed micromorphic model) to describe band-gap phenomena in metamaterials.

1 Introduction

Band-gap metamterials are able to "stop" or "bend" the propagation of waves of light or sound with no energetic cost thanks to their architectured microstructures. Starting from considering an anisotropic unit cell, the global metamaterial can exhibit, at the macroscopic scale, the following behaviors:

- anisotropic behavior with respect to deformation (the deformation patterns vary when varying the direction of application of the externally applied loads),

- anisotropic behavior with respect to band-gap properties (the width of the band-gap varies when varying the direction of propagation of the travelling wave).

In this proceeding, we want to present a continuum model which is able to describe the exotic behavior of metamaterials in a very precise way. Among all possible choices present in the literature, the relaxed micromorphic model [1–8] is the only one which allows us to describe the presence of band-gaps in metamaterials.

We enrich the kinematics of classical elasticity by introducing a tensor field P, known as the micro-distortion tensor field, which adds new degrees of freedom allowing for the description of extra (optic) dispersion curves and thus, for including the effect of microstructure on the dynamical behavior of heterogeneous materials.

The possibility of obtaining a natural generalization of classical elasticity resides in the choice of the constitutive energy densities.

Energy of the relaxed micromorphic model

$$\begin{aligned} J\left(u_{,t}, \nabla u_{,t}, P_{,t}\right) &= \frac{1}{2}\left\langle \rho\, u_{,t}, u_{,t}\right\rangle + \frac{1}{2}\left\langle \mathbb{J}_{\text{micro}} \operatorname{sym} P_{,t}, \operatorname{sym} P_{,t}\right\rangle + \frac{1}{2}\left\langle \mathbb{J}_{c} \operatorname{skew} P_{,t}, \operatorname{skew} P_{,t}\right\rangle \\ &\quad + \frac{1}{2}\left\langle \mathbb{T} \operatorname{sym} \nabla u_{,t}, \operatorname{sym} \nabla u_{,t}\right\rangle + \frac{1}{2}\left\langle \mathbb{T}_{c} \operatorname{skew} \nabla u_{,t}, \operatorname{skew} \nabla u_{,t}\right\rangle , \\ W\left(\nabla u, P, \operatorname{Curl} P\right) &= \underbrace{\frac{1}{2}\left\langle \mathbb{C}_{e} \operatorname{sym}\left(\nabla u - P\right), \operatorname{sym}\left(\nabla u - P\right)\right\rangle_{\mathbb{R}^{3\times 3}}}_{\text{anisotropic elastic - energy}} + \underbrace{\frac{1}{2}\left\langle \mathbb{C}_{\text{micro}} \operatorname{sym} P, \operatorname{sym} P\right\rangle_{\mathbb{R}^{3\times 3}}}_{\text{micro - self - energy}} \\ &\quad + \underbrace{\frac{1}{2}\left\langle \mathbb{C}_{c} \operatorname{skew}\left(\nabla u - P\right), \operatorname{skew}\left(\nabla u - P\right)\right\rangle_{\mathbb{R}^{3\times 3}}}_{\text{invariant local anisotropic rotational elastic coupling}} + \underbrace{\frac{\mu\, L_c^2}{2}\left\langle \operatorname{Curl} P, \operatorname{Curl} P\right\rangle_{\mathbb{R}^{3\times 3}}}_{\text{curvature}} \end{aligned}$$

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with

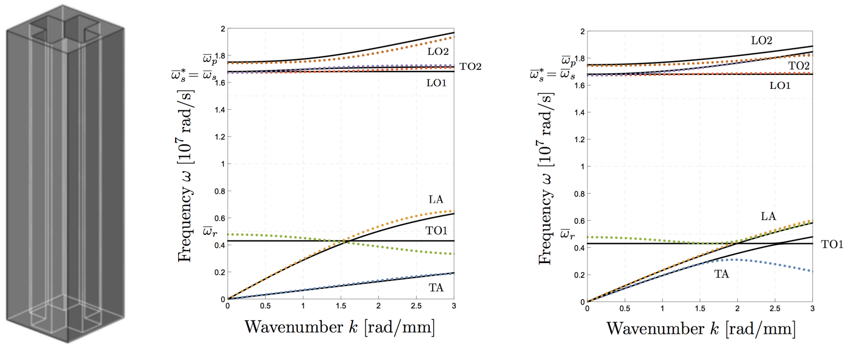
$$\begin{cases} \rho : \Omega \to \mathbb{R}^+ & \text{macro-inertia mass density,} \\ \mathbb{J}_{\text{micro}} : \text{Sym}\,(3) \to \text{Sym}\,(3) & \text{classical } 4^{th} \text{order free micro-inertia density tensor,} \\ \mathbb{T} : \text{Sym}\,(3) \to \text{Sym}\,(3) & \text{classical } 4^{th} \text{order gradient micro-inertia density tensor,} \\ \mathbb{J}_c, \mathbb{T}_c : \mathfrak{so}\,(3) \to \mathfrak{so}\,(3) & 4^{th} \text{order coupling tensors with 6 independent components,} \\ \mathbb{C}_e, \mathbb{C}_{\text{micro}} : \text{Sym}\,(3) \to \text{Sym}\,(3) & \text{classical } 4^{th} \text{order elasticity tensors with 21 independent components,} \\ \mathbb{C}_c : \mathfrak{so}\,(3) \to \mathfrak{so}\,(3) & 4^{th} \text{ order coupling tensors with 6 independent components,} \end{cases}$$

and L_c is the characteristic length of the relaxed micromorphic model. The derived strong equations are

$$\begin{aligned} \rho\, u_{,tt} - \text{Div}\left[\mathbb{T}\,\text{sym}\,\nabla u_{,tt} + \mathbb{T}_c\,\text{skew}\,\nabla u_{,tt}\right] &= \text{Div}\left[\mathbb{C}_e\,\text{sym}\,(\nabla u - P) + \mathbb{C}_c\,\text{skew}\,(\nabla u - P)\right], \\ \mathbb{J}_{\text{micro}}\,\text{sym}\,P_{,tt} &= \mathbb{C}_e\,\text{sym}\,(\nabla u - P) - \mathbb{C}_{\text{micro}}\,\text{sym}\,P - \mu\, L_c^2\,\text{sym}\,\text{Curl}\,\text{Curl}\,P \\ \mathbb{J}_c\,\text{skew}\,P_{,tt} &= \mathbb{C}_c\,\text{skew}\,(\nabla u - P) - \mu\, L_c^2\,\text{skew}\,\text{Curl}\,\text{Curl}\,P. \end{aligned}$$

2 Numerical results

The pertinence of the proposed model is displayed in the dispersion curves, which are in good agreement with their counterparts, obtained by means of the Bloch-Floquet analysis (see Figure 1).



(a) (b) (c)

Fig. 1: Comparison between our relaxed micromorphic continuum model and the COMSOL® one (Bloch-Floquet analysis). In (a) we can see the microstructure considered to generate the infinite periodic metamaterial. In (b) we plot the dispersion branches for $\widehat{\boldsymbol{k}} = (1, 0, 0)$ and in (c) for $\widehat{\boldsymbol{k}} = (\sqrt{2}/2, \sqrt{2}/2, 0)$. Dotted lines represent COMSOL® dispersion curves, continuous lines represent the dispersion curves obtained with the relaxed micromorphic model. The two directions $\widehat{\boldsymbol{k}}$ are used in the fitting procedure.

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