

Constitutive modeling and FEM for a nonlinear Cosserat continuum

I. Münch^{*1}, W. Wagner¹, and P. Neff²

¹ Universität Karlsruhe (TH), Institut für Baustatik, Kaiserstraße 12, 76131 Karlsruhe, Germany

² Technische Universität Darmstadt, FB Mathematik, Schlossgartenstraße 7, 64289 Darmstadt, Germany

The choice of non-standard material parameters in the presented micropolar continuum model leads to a highly nonlinear problem which calls also for a nonlinear numerical treatment. The model benefits in describing length scale effects as really non-linear effects. The numerical torsion test is one example for the possibilities of our model.

© 2006 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

1 Introduction

The Cosserat model includes in a natural way size effects, i.e. small samples behave stiffer in comparison to large samples. These effects have recently received new attention in conjunction with nano-devices. In this context we investigate a geometrically nonlinear Cosserat continuum. We adjust the material parameters of our model in such a way, that microrotations $\bar{\mathbf{R}} \in \text{SO}(3)$ are really independent of the deformation gradient $\mathbf{F} = \nabla\varphi$. Besides the nonsymmetric strain measure $\bar{\mathbf{U}} = \bar{\mathbf{R}}^T \mathbf{F} - \mathbb{1}$, which leads to an internal energy W_{mp} , we include a second strain measure, based on gradients of the microrotation field $\bar{\mathbf{R}}$. This strain measure represents in fact the curvature of the substructure and corresponds to a nonlinear internal curvature energy W_{curv} . In this setting the continuum model can benefit from the microrotation field $\bar{\mathbf{R}}$ by a better description of lower energy modes. Thus, non-classical deformation modes are possible and appear in our analytical and numerical investigations. We define now a strain energy based on $(\bar{\mathbf{U}} - \mathbb{1})$

$$W_{\text{mp}} := \mu \|\text{sym}[\bar{\mathbf{U}} - \mathbb{1}]\|^2 + \mu_c \|\text{skew}[\bar{\mathbf{U}} - \mathbb{1}]\|^2 + \frac{\lambda}{2} (\text{tr}[\bar{\mathbf{U}} - \mathbb{1}])^2 \quad . \quad (1)$$

From the classical tension test and its deformation mode (uniaxial strain) the Lamé constants μ and λ for the symmetric and volumetric part are uniquely determined. However, it is not clear how to choose the so called Cosserat couple modulus μ_c . It is necessary to activate rotational modes in deformation to investigate the influence of μ_c . Therefore, a torsion test is more appropriate than a shear test. Thus, we consider the fully three-dimensional framework in our numerical formulation. At this stage, we can discuss the influence of μ_c . It penalizes skewsymmetric parts of $\bar{\mathbf{U}}$ and for $\mu_c \rightarrow \infty$ we expect $\bar{\mathbf{U}} \in \text{Sym}$. As an obvious consequence $\bar{\mathbf{R}} \rightarrow \text{polar}[\mathbf{F}]$, where $\text{polar}[\mathbf{F}]$ is the orthogonal part in the polar decomposition of \mathbf{F} . Our investigations have shown that already for $\mu_c \approx \mu$ the microrotation field $\bar{\mathbf{R}}$ represents not really an independent quantity but the microrotations equal nearly the polar decomposition of the deformation gradient \mathbf{F} . Thorough analytical investigations in [3] have shown, that small or vanishing values for μ_c can lead to complex but energetically favourable solutions which may reflect deeper aspects of material behaviour. The Cosserat response is determined by simultaneously minimizing the total energy (strain and curvature) $W_{\text{mp}} + W_{\text{curv}}$ w.r.t. deformations and microrotations. Here we use $W_{\text{curv}} := \frac{\mu}{q} (1 + L_c^2 \|\mathfrak{K}\|^2)^{\frac{q}{2}}$, $\mathfrak{K} = \bar{\mathbf{R}}^T D_x \bar{\mathbf{R}}$, $q > 3$. Curvature energy acts like torsional springs within the material and influences angular momentum. The additional phenomenological material parameter L_c represents the stiffness of these intended torsional springs. Note that for $L_c \rightarrow \infty$, $\mu_c = 0$ and appropriate boundary conditions, the model behaves like a classical linear St.Venant-Kirchhoff material. On the other hand (see Fig.1) we can identify a set of material parameter μ_c and L_c , which describes rather a material of Neo-Hooke type.

2 Numerical torsion test

We use a standard finite element formulation with 8 or 27 node brick elements. The same shape functions are chosen for displacement and rotational fields. A multiplicative update of the micro-rotational field is done by using quaternions as suggested in [4]. One of our numerical tests is a torsion test of a clamped beam with torque load on the tip. The beam has a length $l = 2$, a cross section $a = b = 0.4$ and an elastic modulus $E = 1 \cdot 10^6$ together with the Poisson ratio $\nu = 0.3$. There are no boundary conditions on microrotations but on displacements at lower and upper border. The associated load-displacement behaviour is plotted as torque over twist of the tip in Fig.1. The 'Neo-Hooke' curve represents a classical solution. For $\mu_c = \mu$ the Cosserat solution shows from the beginning of deformation nearly unbounded stiffness for $L_c \rightarrow \infty$. Only for $L_c \rightarrow 0$ it is possible to simulate a classical solution, but in this case the model does not really include curvature effects such

* Corresponding author: e-mail: im@bs.uka.de, Phone: +49 721 608 2289, Fax: +49 721 608 6015

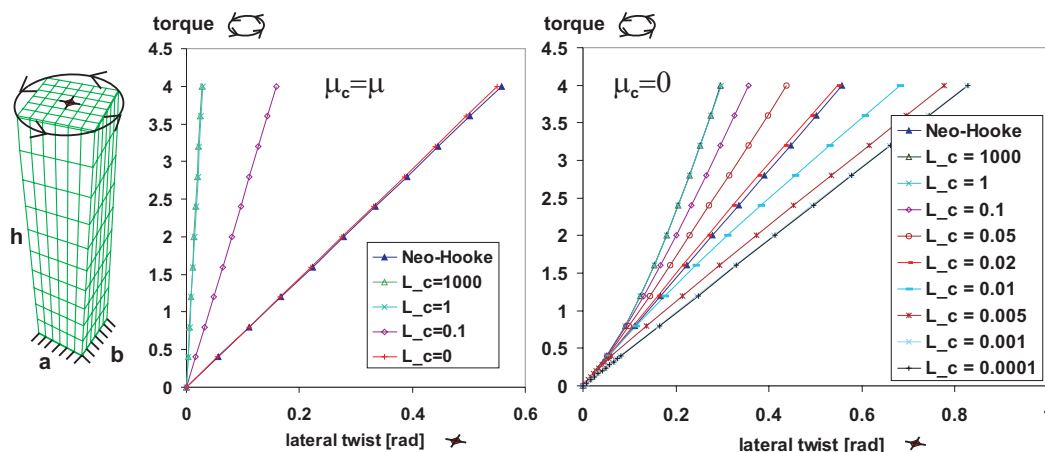


Fig. 1 Torque-twist diagrams with $\mu_c = \mu$ (left) and $\mu_c = 0$ (right) for various internal length scale factors L_c .

as size-dependence. A completely different behaviour appears for $\mu_c = 0$ in which case it is possible to simulate classical infinitesimal elasticity for any values of the internal length scale factor L_c under small loads while length scale effects arise under higher loads. For various L_c the presented mechanical model offers stronger **and** weaker response than the elastic model based e.g. on the Neo-Hooke energy with similar elastic moduli when subjected to larger loads. Note, that the variation of the internal length scale L_c is equivalent to the modification of sample sizes. It can also be seen in Fig. 1 left, that the stiffness response possesses an upper and lower limit for any internal length scale factor L_c . The bounded upper limit of the initial tangential stiffness seems of physical importance because this means bounded stiffness for arbitrary small samples which is a fundamental physical requirement [5].

Due to the separation of symmetric and skewsymmetric parts in the strain energy Eq. 1 the need for identification of μ_c appears. Torsion tests are best suited for investigations concerning the effects of μ_c and L_c because of marked rotations and gradients of rotations in deformation. The problem for $\mu_c = 0$ is mathematically well posed (existence of minimizers) as it has been shown in [6]. In the proposed framework with $\mu_c = 0$ the linearized Cauchy-stress tensor remains **symmetric** in contrast to traditional Cosserat-type models. Such an approach is possible only in a geometrically nonlinear treatment. However, since we deal with an overall nonlinear, non-convex two-field problem, computed equilibrium solutions may lose their stability since a highly complicated energy landscape occurs. The pseudo-homogenization achieved through the micropolar continuum theory does not represent immediately a real physical material. By considering an imperfection field critical states of stability can be excluded.

References

- [1] P. Neff, Finite multiplicative elastic-plastic Cosserat micropolar theory for polycrystals with grain rotations. Modelling and mathematical analysis, *to appear in Int. J. Eng. Sci.*, Preprint 2297 <http://wwwbib.mathematik.tu-darmstadt.de/Math-Net/Preprints/Listen/pp03.html>
- [2] P. Neff, S. Forest, A geometrically exact micromorphic model for elastic metallic foams accounting for affine microstructure. Modelling, identification of moduli and computational results, Preprint 2373 <http://wwwbib.mathematik.tu-darmstadt.de/Math-Net/Preprints/Listen/pp04.html>
- [3] P. Neff, I. Münch, Simple glide for a nonlinear Cosserat model: Analytical and computational results with induced microstructure, Preprint 2414 <http://wwwbib.mathematik.tu-darmstadt.de/Math-Net/Preprints/Listen/pp04.html>
- [4] C. Sansour, W. Wagner, Multiplicative updating of the rotation tensor in the finite element analysis - a path independent approach, *Comp. Mech.* **31**, 153-162, (2003).
- [5] P. Neff, The Cosserat couple modulus for continuous solids is zero viz the linearized Cauchy-stress tensor is symmetric, *to appear in Z. Angew. Math. Mech.*, Preprint 2409 <http://wwwbib.mathematik.tu-darmstadt.de/Math-Net/Preprints/Listen/pp04.html>
- [6] P. Neff, Existence of minimizers for a finite-strain micromorphic elastic solid, *to appear in Proc. Roy. Soc. Edinb. A*, Preprint 2318 <http://wwwbib.mathematik.tu-darmstadt.de/Math-Net/Preprints/Listen/pp04.html>