FETI-DP Methods for P-Elasticity

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A FETI-DP method is introduced for the problem of linear P-elasticity which arises from linear elasticity by the introduction of a matrix P and which is motivated by micromorphic models. Numerical results as well as a condition number estimate are presented.

1 Introduction

We consider the variational problem

$$\min_{\varphi}\left(\int_{\Omega}\mu_e||\text{sym}(P^{-1}F-\text{Id})||_F^2+\frac{\lambda_e}{2}\text{trace}(\text{sym}(P^{-1}F-\text{Id}))^2\ d\lambda\right), \quad (1)$$

where $F=\nabla\varphi\in\mathbb{R}^{3\times 3}$ and φ is the deformation, i.e., $\varphi(x)=x+u(x)$ with the displacement $u(x)$. The constants μ_e and λ_e are the Lamé constants which depend on the Poisson ratio ν and the Young modulus E. In contrast to classical linear elasticity a matrix $P\in\text{GL}(3)$, the group of all real invertible 3×3 matrices, is introduced. Here, $P=P(x), x\in\Omega$, is a micromorphic field which usually is not of gradient structure, i.e., P does not necessarily have a potential. If P is the identity, (1) reduces to the standard formulation of linear elasticity. Due to the considered model the matrix P can either be used to describe small scale material oscillations superposed on the macroscopic deformation φ or it is the plastic distortion in a multiplicative decomposition of the deformation gradient F; cf. [1].
Since we use FETI-DP methods to solve the variational problem, we have to derive the weak formulation and discretize it by low order h-finite elements ($\mathcal{P}_2$).

2 Condition number estimate

Since FETI-DP is a preconditioned conjugate gradient method, we are interested in an estimate of the condition number of the preconditioned system matrix; cf. [2]. For this result we have to assume that the matrix P has gradient structure, i.e., there exists a function $\psi:\Omega\subset\mathbb{R}^3\to\mathbb{R}^3$ such that $P=\nabla\psi$, and that we have a well chosen set of primal variables consisting of vertex and edge average constraints; for further details, see [2]. Under certain assumptions on the triangulation and the domain decomposition we can prove that there exists a positive constant C such that

$$\kappa(M^{-1}F)\leq C\left(1+\log\left(\frac{H}{h}\right)\right)^2, \quad (2)$$

with the standard FETI-DP preconditioner M^{-1} and the system matrix F. In (2) $\frac{H}{h}$ is defined as $\max_i\frac{H_i}{h_i}$, where H_i is the diameter of the subdomain Ω_i and h_i is the average element diameter in the subdomain.

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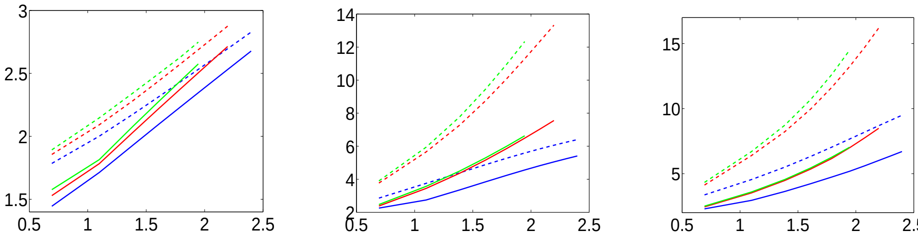
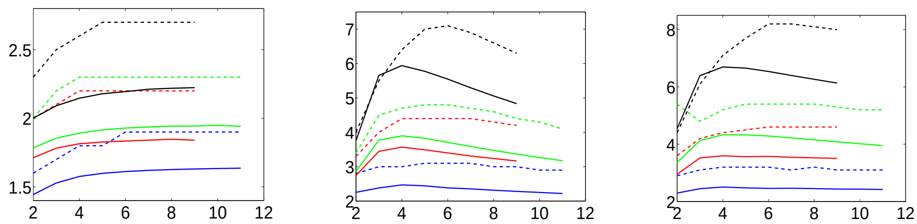
3 Numerical results

For our numerical experiments we choose $P = \nabla\psi$ and on the Dirichlet boundary we assume $\varphi(x) = \psi(x)$. Due to the variational formulation in (1) the solution $\varphi(x)$ is known to be $\psi(x)$ for every $x \in \Omega$. We assume a homogeneous material with $E = 210$ and $\nu = 0.29$. For the experiments we use different sets of primal variables and quadratic nodal basis functions. The computations are carried out on an Opteron cluster with 8 dual processor nodes with 2.2 Ghz and 4 GB each.
For the computations we choose different $\psi(x)$ and hence different matrices P: $\psi_1(x)$ refers to a linear increasing twist around the z-axis, $\psi_2(x)$ and $\psi_3(x)$ both belong to a transformation of the unit cube into a spherical dome with different thickness and angular values. The results we are interested in are the behavior of the condition number or the maximum eigenvalue, respectively, since the minimum eigenvalue is 1, and the behavior of the number of iterations. We consider two different cases. In the first case we keep the number of subdomains fixed , i.e., $\frac{1}{H}$ = const., and increase the size of the subdomain, i.e., $\frac{H}{h}$. From the condition number estimate we would expect a linear behavior of the square root of the maximum eigenvalue with increasing $\log(\frac{H}{h})$. Hence in the following figures the square root of the maximum eigenvalue is plotted against $\log(\frac{H}{h})$ for ψ_1, ψ_2, and ψ_3. The dashed lines are results for only edge averages as primal variables and the solid lines for edge average and vertex constraints, the different colors represent the different $\frac{1}{H}$ values (blue: 2, red: 3, green: 4).



In the second case we consider a fixed size of the subdomain, i.e., $\frac{H}{h}$ = const. Then we would expect our maximum eigenvalue and the number of iterations to be constant when we increase the number of subdomains, i.e., $\frac{1}{H}$. Therefore, one figure for each ψ is presented. The solid lines are the square roots of the maximum eigenvalues and the dashed lines the iteration numbers divided by 10, both plotted against $\frac{1}{H}$. The different colors represent different sets of primal variables and sizes of subdomains (blue: edges & vertices, H/h=2; green: only edges, H/h=2; red: edges & vertices, H/h=3; black: only edges, H/h=3).



References

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