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Micromechanical Modeling of Woven Fiber Composites for the Construction of Effective Anisotropic Polyconvex Energies

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In computer simulations where constitutive equations are considered polyconvex energies can preferably be used because the existence of minimizers is then automatically guaranteed. In this work we investigate the capability to simulate woven fiber composites using polyconvex energies. A virtual experiment of the microstructure of the considered composite is performed and the results are used for the specification of an effective macroscopic anisotropic polyconvex model.

1 Introduction

Due to their light weight, high stiffness, strength and toughness woven fiber composites are becoming increasingly important in many engineering applications ranging from roof constructions to weather-proof awnings. In numerous cases special materials come to operation, which consist of a woven fiber network made of e.g., glass-, textile- or synthetic fibers, embedded in a silicone-, polymer- or rubber-like matrix. For this reason a highly non-linear material behavior with irreversible effects and, due to the fiber-reinforcement of the material, strong anisotropy effects have to be taken into account. For an optimal design of such constructions a prediction of their stress-strain behavior is required.

We concentrate on a phenomenological description of a class of textile composites in which two fiber families are aligned in mutually orthogonal directions, the warp and fill direction, and restrict ourselves on finite elasticity. To derive a homogenized macroscopic model we focus -as a first step- on the performance and the results of a biaxial tension test of a microstructure. Each component, the two fiber families and the matrix, are here assumed to be individually isotropic, hyperelastic materials and therefore described by isotropic polyconvex energies. The polyconvexity condition, introduced by Ball [1], plays a significant role in finite elasticity, because by satisfying this condition the Legendre-Hadamard ellipticity and also (together with the fulfillment of the coercivity condition) the existence of minimizers is ensured. Finally, we adjusted the overall stress-strain curves obtained for the heterogeneous microstructure to an orthotropic polyconvex model for an assumed homogeneous macrostructure. The anisotropic polyconvex energy function is taken from [5] based on specific orthotropic structural tensors. The first general formulation of orthotropic polyconvex functions has been proposed in [4], which can be seen as an extension of the transversely isotropic polyconvex energies derived in [3]. To account for the condition of a stress-free reference configuration a priori, an attractive approach is given in [2].

2 Material Modeling

Taking the mutually orthogonal fiber-reinforcement of the material into account, we assume an additive decoupling of the energy into an isotropic part ψ^{iso} for a weak matrix and a part $\tilde{\psi}$ for the embedded fibers. Then the energy reads

$$\psi = \psi^{iso}(I_1, I_2, I_3) + \tilde{\psi}(J_4, J_5, I_3)\,, \quad \text{with} \quad \psi^{iso} := \delta_1\, I_1 + \delta_2\, I_2 + \delta_3\, I_3 - (2\,\delta_1 + 4\,\delta_2 + 2\,\delta_3) \ln\sqrt{I_3}\,, \quad (1)$$

with the three principal invariants of the right Cauchy-Green deformation tensor $\boldsymbol{C} := \boldsymbol{F}^{\boldsymbol{T}}\boldsymbol{F}$, i.e., $I_1 = \mathrm{tr}[\boldsymbol{C}]$, $I_2 = \mathrm{tr}[\mathrm{Cof}\boldsymbol{C}]$, $I_3 = \det[\boldsymbol{C}]$, where $\boldsymbol{F}$ is the deformation gradient. Furthermore the mixed invariants $J_4 = \mathrm{tr}[\boldsymbol{C}\boldsymbol{G}]$ and $J_5 = \mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}]\boldsymbol{G}]$ in terms of the specific structural tensor $\boldsymbol{G}$ proposed in [5] are considered. In detail, for the fibers we choose the function

$$\tilde{\psi} := \xi \left(\frac{1}{g^\alpha(\alpha+1)} J_4^{(\alpha+1)} + \frac{1}{g^\beta(\beta+1)} J_5^{(\beta+1)} + \frac{g}{\gamma} I_3^{-\gamma} \right)\,, \quad \text{with} \quad g := \mathrm{tr}\boldsymbol{G}\,, \quad (2)$$

a general proof of the polyconvexity of the individual functions J_4^k and J_5^k for $k \geq 1$ in terms of symmetric, positive definite structural tensors $\boldsymbol{G}$ is given in [5] and specifications for orthotropy can be found in [4] as well as in [2]. It should be remarked that if $\boldsymbol{G}$ is equal to the second-order identity tensor $\mathbf{1}$, the function (2) is isotropic, otherwise if $\boldsymbol{G}$ is equal to a specific structural tensor, we obtain an anisotropic function (2). In order to ensure the polyconvexity of (1) the material

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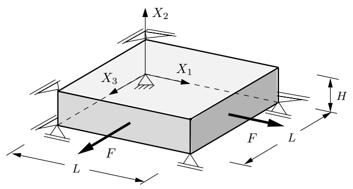
parameters have to satisfy the restrictions $\delta_1,\ \delta_2,\ \delta_3,\ \alpha,\ \beta,\ \xi \geq 0, \gamma \geq -\frac{1}{2}$; for the proof of coercivity of (1) see [5]. The stress-free reference configuration condition of the the stresses at natural state, i.e., $\boldsymbol{S}(\boldsymbol{C} = \boldsymbol{1}) := 2\partial_C \psi(\boldsymbol{C} = \boldsymbol{1}) = \boldsymbol{0}$, is a priori fulfilled.

3 Virtual Experiment

We simulate a force-driven biaxial tension test of a microstructure of a woven fiber composite depicted in Fig. 1a. The length and height are set to $L = 30$mm, $H = 15$mm. The microstructure is discretized by 7796 ten-noded tetrahedral finite elements. Each component, i.e.,the fibers in warp-direction (w) and the fibers in fill-direction (f) as well as the matrix (m), is described by isotropic polyconvex energies (1) with $\boldsymbol{G} = \boldsymbol{1}$ given by

$$\psi^w_{iso} = \psi^w(\boldsymbol{C}, \boldsymbol{1})\,, \quad \psi^f_{iso} = \psi^f(\boldsymbol{C}, \boldsymbol{1})\,, \quad \psi^m_{iso} = \psi^m(\boldsymbol{C}, \boldsymbol{1})\,, \tag{3}$$

with the material parameters presented in Fig. 1b. The normal displacements u_1 and u_3 of the load-contact surfaces (Fig. 2a) of the microstructure are linked. The biaxial tension leads to a distribution of the Cauchy stresses $\boldsymbol{\sigma} = \det\boldsymbol{F}^{-1}\boldsymbol{F}\boldsymbol{S}\boldsymbol{F}^T$ in

a. 

b.

	δ_1	δ_2	δ_3	ξ	α	β	γ
w	313.36	0.001	0.0002	1.0	0.002	38.587	−0.50
f	−	−	−	1.0	62.66	0.0008	−0.50
m	−	−	−	1.0	5.00	0.0008	−0.50
	[N/mm^2]				[−]		

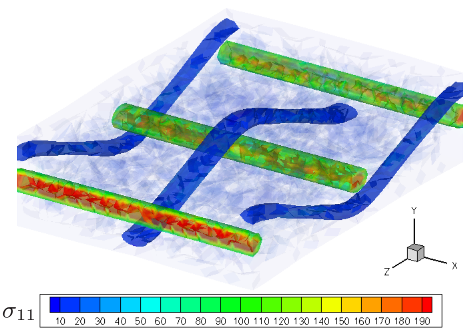
Fig. 1 Biaxial tension test of microstructure: a. Boundary conditions, b. Material parameter set.

x_1-direction, which is shown in Fig. 2a., and to an assumed overall stress-strain-behavior of the microstructure, presented as red-coloured curves in Fig. 2b. Here the strong anisotropy effect of the fiber-reinforcement of the material can be immediately noticed, since the first Piola-Kirchhoff stresses $\boldsymbol{P} = \boldsymbol{F}\boldsymbol{S}$ in the considered directions enormously differ from each other. As an effective macroscopic model also the energy given by (1) is chosen; the anisotropy is incorporated by inserting an orthotropic structural tensor into (1), i.e.,

$$\psi_{ortho} = \psi(\boldsymbol{C}, \boldsymbol{G}^{ortho})\,, \quad \text{with} \quad \boldsymbol{G}^{ortho} = \text{diag}(a, b, c) \quad \text{and} \quad a, b, c > 0\,. \tag{4}$$

This orthotropic model, depicted as black-coloured curves in Fig. 2b, is finally adjusted to the " experimental " curves; the material parameters of the macroscopic anisotropic model are identified as follows

$$\begin{aligned} &a = 4.537,\ b = 2.838,\ c = 1.853,\ \delta_1 = 0.0043\ \text{N/mm}^2,\ \delta_2 = 0.0014\ \text{N/mm}^2, \delta_3 = 0.0061\ \text{N/mm}^2, \\ &\alpha = 26.246,\ \beta = 13.646,\ \gamma = -0.09,\ \xi = 1.0\ \text{N/mm}^2\,. \end{aligned} \tag{5}$$

a. 

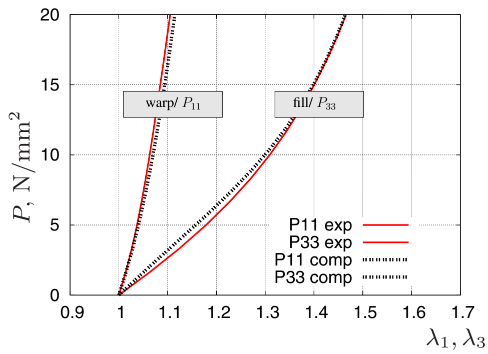
b. 

Fig. 2 Biaxial tension test of microstructure: a. Distribution of Cauchy stresses in x_1-direction, b. Fitting of macroscopic model: First Piola-Kirchhoff-stresses in x_1- and x_3-direction $(P_{11},\ P_{33})$ vs. stretches $\lambda_1 = 1 + u_1/L$ and $\lambda_3 = 1 + u_3/L$, respectively.

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