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Back to the Classics: R-Evolution Towards Statistical Equilibria

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Back to the Classics: R-Evolution Towards Statistical Equilibria

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Abstract

Economic modeling struggles often with a lack of realism. The reason for that is that economic theory for the last 100 years has focused on simplifying assumptions which reduced important aspects of the economic reality. Concentrating on fix-point solutions and external statistical shocks prevented the profession from accurately describing the economy as a complex system with characteristics like feedback mechanism, evolution, and emergence. We propose a re-evaluation of major findings in the Classical economic literature. The classical literature described the economic system as inherently probabilistic. In this spirit, we discuss the importance of statistical equilibrium models as one way to model complex economic systems in a probabilistic way.

JEL Code: B12, B41, C18, D59

1 Introduction

Economics scientific field open to disputes and discussions about the material circumstances of life. The different schools of thought compete for academic attention, with different assumptions and implications regarding socio-economic circumstances. The neoclassical paradigm remains the leading ideology in the academic debate. This hegemony is repeatedly questioned when economic crises occur (Krugman 2009, Varoufakis et al. 2012, Chorafas 2013). However, neoclassical thinking remains deep in the repertoire of economists.

We criticize one aspect of the neoclassical paradigm. We criticize the use of the neoclassical general equilibrium theory as a methodologically flawed concept. We discuss the prevailing mindset in economics that the actions and circumstances of agents are deterministic in their nature. This leads to deterministic fix-point solutions which quite often differ from economic reality. We propose a methodological solution to this. Statistical equilibria are in their nature probabilistic and instead of focusing on one solution, its solution is a distribution. The distributional form has the advantage of assigning a probability to realizations and links the economic profession back to its roots: The Political Economy approach of the Classical authors.

We decided to structure this paper in a way that opens the discussion about equilibrium, especially for economists who experienced neoclassical training. In the first step, we mirror the teaching approach in most economic classes by focusing on the neoclassical perspective of discussing equilibrium. We discuss four important characteristics that are implied in the neoclassical equilibrium theory. We contrast those four characteristics of complexity theory and how they relate to economic activity. In the second step, we derive a complex understanding of economic activity independent of neoclassical theory. We give a detailed description of statistical equilibria and probabilistic thinking and accentuate those thoughts with references to classical political economics. We use the Quantal Response Statistical Equilibrium (QRSE) model (Scharfenaker & Foley 2017) as one of many elaborate models to analyze economic activity in a probabilistic framework.

Our contribution to the literature is two-fold. On one hand, we provide an introduction to probabilistic thinking as a way to better understand economic activity. To make probabilistic economics accessible to a wide audience, we first contrast it to the established neoclassical framework and secondly, derive it independently of the neoclassical framework. On the other hand, we show that the statistical equilibrium approach is an adequate way to combine Classical Political Economics with probabilistic thinking.

2 The Neoclassical Equilibrium

The neoclassical theory has a different understanding of equilibrium compared to the classics. It breaks in its theoretical understanding and conceptual framework with the earlier understanding of economics. Cournot (Mirowski 1991, p. 210) used optimization theory, in the form of Lagrangian and Hamiltonian, to determine the maximal outcome in an economy. The mathematical approach of Cournot threw the shadows of the marginalist revolution ahead. The “premier troika of the marginalist revolution; Leon Walras, William Stanley Jevons, and Carl Menger” (Mirowski 1991, p. 254) pushed the use of mathematical equations further. There exists a similarity between the equations in economics and those used in Newtonian classical mechanics (Pareto & Priuli 2012, p. 16).

To benefit from differential and integral calculus and further developments from Newtonian and Leibnizian methods, neoclassical economics incorporated equilibrium theory deep into its methodological framework and established itself as the dominant paradigm. The traditional neoclassical implies several assumptions, a few of which necessarily lead to specific outcomes. We focus on perfect rationality, homogeneous agents, the deterministic and stochastic nature of the models, and the existence of fix-point equilibria as four of these implicit assumptions. We discuss those four assumptions with respect to their implicit statements and their limiting impact on the nature of modeling.

From a philosophy of science standpoint, there are some features that provide some explanation for the dominance and survival of the neoclassical paradigm, even under very unrealistic assumptions. Friedman (1953, p. 14) highlights how important it is for a theory that “it ‘explains’ much by little”. He describes the necessity to conceptualize the complex nature of the economy in simple terms as the “wildly inaccurate descriptive representations of reality [that] abstracts the common and crucial elements from the mass of complex and detailed circumstances”(p. 14).

Four characteristics of the neoclassical theory stand out in their importance for the methodological framework of the theory. (1) Perfect rationality of agents and (2) homogeneity of the agents are necessary to achieve results in the applied mathematical concepts. The (3) exogenous shocks through deterministic/stochastic error terms will lead to (4) fix-point solutions of the model. These four aspects are important aspects of the neoclassical theory and characterize the understanding of equilibrium in that conceptual framework.

Perfect rationality is expressed by all agents in the neoclassical framework. Perfect rationality implies that agents experience specific skills, that allow the agent to make decisions that are perfect. Every agent is perfectly aware of their own costs and benefits for each transaction. The agents are perfectly aware of all the actions of potential counterparts in that transaction. This perfect insight into the current market environment allows the agents to calculate the outcome of each action and choose the most beneficial. The perfect rationality of agents is not limited to the current time interval but stretches infinitely into the future. Perfect rationality excludes any surprises for the agents as they are able to calculate the optimal solution for themselves throughout time. For economists, it is possible to calculate today all infinite payouts.

Homogeneous agents are another important concept in neoclassic economics. It is assumed that agents have identical interests, costs, and payouts. Such an assumption is justified through the existence of perfect competition where any divergence in the action of an agent would either assign the whole market to a single agent (underbidding the competitors) or deny any market share to that agent (overcharging). These homogeneous agents allow a researcher to accumulate the whole economy into one single agent. This representative agent simplifies the economic analysis as it makes it irrelevant if a model consists of one representative agent or several identical agents. The idea of homogeneous agents bypasses the necessity to analyze distributional effects as all agents in the economy are identical (Kirman 1992).

Shocks or crises in the neoclassical framework are temporary exogenous and not a result of internal model dynamics. The neoclassical framework rules out that endogenous dynamics lead to bubbles or misvalidation of assets or transactions. Any non-rational action of agents would lead to arbitrage which other agents would immediately realize and use to increase their own payout stream. The exogenous shocks are independent of the activity of the agents. They must be defined by the researcher and are normally characterized by normal-distributed error terms. The activity of agents and how they adjust to external shocks become the focus of economic analysis. After an external shock, the system recalibrates to a new equilibrium and remains in that state until a new shock disrupts the optimized plans of the agents.

The neoclassical approach relies on fixed-point theorems. The solutions of a system with a supply function and a demand function, q^* and p^*, are considered fix-points or quantity and price equilibria resulting from the balance between supply and demand[1]. The theory states that for any market, prices would be the same. For the price of a commodity category to have the same numerical result, the distribution of prices would be a Dirac Delta distribution. It is disputed if the Dirac Delta distribution contains the necessary characteristics to be classified as a distribution. The Dirac Delta distribution contains all observations at one single value, like a histogram with only one bar. The Dirac Delta distribution has zero entropy. Zero entropy implies that there is no uncertainty for an agent. If an agent switches between states, those switches do not have any uncertainty or slow adjustment process. The behavioral temperature is marginally close to zero (Foley 2020*b*).

Neoclassical economics has adapted and evolved on those core features. Modern approaches often include several agents with different strategies and limited foresight. These agents can err as the error term is not applied to the external conditions but to the internal calculation of the agents. Those adaptions to the model can lead to several different optimal strategies for the agents and solutions to the model, including bifurcating paths. This does not, however, change the underlying criticism of the approach. The approach continues to build on the old paradigm as the assumptions and modeling techniques are closely related. It remains that the exogenous fluctuation of the model is the only source of derivation from the fix-point solution. The lack of internal uncertainty in the model creates a world that is easy to analyze but lacks specific features that are characteristic of social systems. The lack of uncertainty about the outcome creates a world of zero informational entropy.

[1] The point where supply and demand intersect defines the fixed-point equilibrium in that market, resulting in an ordered pair of equilibrium quantity and price. This single number would represent the price for every transaction in that market. In reality, however, prices vary.

Both Walras and Marshall created an abstract representation of markets (Foley 2020*a*). They substituted the distribution of transaction prices by the mean of transaction prices. This creates the possibility of disequilibrium prices. As markets are devices that generate equilibrium, no transaction would occur at disequilibrium prices. One methodological solution to avoid disequilibrium prices is a fictional auctioneer. This Walrasian auctioneer eliminates transactions at those "disequilibrium prices" and leads to a general equilibrium. This Walrasian auctioneer calls the prices until the total excess of demand is equal to zero. If the relative prices vector does not clear all markets, another round takes place. This is repeated until a price vector is found which clears all markets at the same time. The prices are single numbers that represent the mean. In seeking an abstract representation of markets, both Walras and Marshall resorted to the mathematical simplification of representing the statistical distribution of transaction prices by its mean, presumably with the idea that the equilibrium statistical distribution of transaction prices is highly concentrated so that the mean would closely approximate the actual distribution (Petri & Hahn 2002). The Walrasian understanding of markets can be conceptualized in terms of statistical mechanics. When the market is thought of a probability field of transactions[2]. In this probability field, non-equilibrium prices are possible. The existence of non-equilibrium prices creates the possibility that identical agents *ex ante* receive different bundles at different prices in the exchange (Scharfenaker & Yang 2020).

3 Complexity Theory

It is common to analyze complex dynamics in the context of biological or physical systems. Complexity research has first been adapted to artificial societies and later to real-world social systems (Mitchell 2011). The goal of Complexity Economics is to model and analyze social systems as complex systems to gain additional insight into the underlying system which is not able to be discovered in other frameworks. Complexity economics is often criticized for its 'mathiness' (Romer 2016), while it relies in its model setup on simplified descriptions of human behavior, rules of thumb, or habits (Arthur 2021). Assumptions in a complex system framework can appear ad-hoc and must therefore properly defined to maintain academic standards. Similar to the premise of the neoclassical paradigm, these assumptions must not be explicitly modeled but they are the underlying foundation of complex models.

The methods used in complexity theory are manifold and diverse. They stretch from agent-based modeling, qualitative reasoning, network theory, and information theory. These approaches take different points of theoretical departure. The agent-based approach follows a strict micro-foundation while network theory comes from a more macro perspective (Miller & Page 2007). What all of these approaches have in common is that they are able to model the system that they analyze as an (1) evolving and (2) emerging system with (3) heterogeneous agents that face (4) bounded rationality. While complex

[2]Transactions have a probability to occur. This probability can depend on endowments, locally observed prices, or other factors (Epstein et al. 1997).

systems can have various characteristics, much like neoclassic economics models, we focus on those four aspects. We discuss the concepts of evolution, emergence, bounded rationality, and heterogeneous agents. We contrast how they relate to neoclassical economics and are by themselves realistic characteristics of socioeconomic systems.

Evolution conceptualizes how agents learn by experience, both from others' experiences and from their own participation in the system. The exposure to those experiences and interactions makes agents adjust their course of action over time. The learning process for the complexity approach means that the agents endogenously adapt to survive. This contrasts the Neoclassical approach where a game is played with predetermined rules set by the researcher. This removes all pedagogical possibilities for the agents to adapt over time. The strategies of the agents reflect the rules of the game, not their own learning process.

Emergence is an important aspect of complexity theory. It describes when the system expresses characteristics that are unexpected from the original setup or the specifications of the agents. Feedback loops within the system and self-organization are often the reason for emergence. Feedback loops can be positive or negative, where a specific action reinforces or weakens the incentive of such an action. Such feedback loops are of interest when agents act and are analyzed in their environment. In network theory, those loops are named motifs and their large variety of realization can be visually expressed (Shoval & Alon 2010, Cimini et al. 2021). In the case of a negative feedback loop, if all agents perform the same action, let's say *buy*, it becomes more interesting for a single agent to switch their strategy to *sell*. An example of a positive feedback loop could be, despite its negative effects on the system as a whole, the digging of a well. Digging deeper than your neighbor gives you more water but at the same time, it incentivizes the neighbor to dig deeper themselves. In self-organization, local rules will be created on a macro-level pattern or functionality that was not intended or organized by an external planner. A flock of birds is the result of self-organization as individual birds want to fly next to their peers, but not too close to remain in their own personal space (Reynolds 1987). Those two restrictions lead to the behavior of the flock that dances in the skies, without predefined directions or patterns where the behavior of the flock is more than the sum of all birds (Anderson 1972). In social sciences, the division of labor and the rise of society are considered examples of such self-organization (Graeber & Wengrow 2021).

Bounded rationality is a rivaling concept to perfect rationality. Rather than allowing agents to make the perfect decision that maximizes their payoff as they have perfect knowledge across space and time, the capabilities of agents to make decisions are restricted. It explicitly assumes the cognitive, informational, and calculatory limitations of agents in decision-making. Bounded rationality assumes that agents make their decisions based on their guts and the rule of thumb. Repetitively occurring decisions that are of the same or similar nature are not optimized but rather copied from previous instances and, perhaps, mildly adapted. The reason is that the acquisition of information and the (re-)calculation of decisions is too costly compared to the expected improvement of the outcome. In other circumstances, agents simply lack the necessary information to make a well-informed decision. This can be the unknown life expectancy and return on an investment, which forces the agent to take an educated guess and bet on possible outcomes.

The heterogeneity of the agents is an important aspect of modeling and analyzing complex systems. It is the result of the combination of evolution and bounded rationality. Even in a system where all agents start off with the same endowments, preferences, and objective functions, bounded rationality will lead to different perceptions of the environment. Different perceptions of the environment will lead to different strategies and payouts in the next step. The different success of the agents leads to different forms of adaption and evolution, and therefore heterogeneous agents. Even if agents start in the beginning with randomly assigned endowments and skills, the heterogeneity will persist over time as simulations with artificial societies have shown (Epstein et al. 1997).

The constant and consistent evolution, adaption, and emergence of agents and patterns make complex systems an interesting point of study. These can be biological or chemical systems, but also in social systems, we observe such characteristic behavior. These characteristics lead to developments of the system that are not predictable in advance if modeled correctly. This shifts the focus of the analysis from the possible/exact outcome to the mechanism that drives the system. This systemic level describes inherent phenomena and is called meso-level (Schulz-Gebhard 2023). This meso-level as the linkage between the micro and macro level gets the spotlight. In empirical work where individual agents are not observable, it is important to find ways to extract the mechanism on the meso-level, as well as the individual behavioral rules to make statements about the system as a whole.

4 The Classical Equilibrium

The study of the economy by the Classics is, in today's wording, complexity theory. Without naming it, Adam Smith, Karl Marx, and others described the economic process using many features developed by Complexity Theory, such as the understanding of the economy as an evolving and emerging system, consisting of heterogeneous agents with bounded rationality.

The classical theory includes complex system features. This inclusion is visible in the examples of the division of labor (Smith 1976) or the equalization of profit rate (Marx & Korsc 2009). Despite their methodological idiosyncrasies, Smith, Marx, and Ricardo were able to describe the economy in such ways because their theories were reflections of reality, not ideal creations from their minds. Marx's method starts the investigation from the concrete and returns to it after the abstraction process in which he discovers the determinations of the concepts retrieved from material reality itself. Their theory was a mirror of the real world and an attempt to model how agents make decisions and all the institutional constraints that shape each historical phase. For instance, the wealth of nations arises from productivity gains unlocked by the labor division, which results from self-interest pursuit in Smith. The driving force of economic decisions is self-interest, not benevolence for Smith. Agents are actually trying to be better off despite all other agent's decisions.

Going deeper into Classical reasoning, Smith's labor theory of value was built around the recognition of self-interest as a feature of human nature. As humans are prone to exchange, producers will find the best response to be specialization and subsequent exchange of specialized production (Foley 2020*a*). Specialized production unlocks gains in productivity. The increased productivity comes from the increased skill level of the producer who improves their technique through the repetitiveness of the work, the time saved from passing from one activity to another, and facilitating other labor technology development. As the producers evaluate their outcomes by comparing return and costs for producing with other producers, there is a tendency for the equalization of the income-to-effort ratios. The movement of producers in and out of different trades generates a negative feedback process that stabilizes the distribution of income-to-effort ratios around a central location parameter, a natural income-to-effort ratio, with some variability around that.

Marx's critique of Political Economy is an approach to introduce the Classical theory. His work reveals the laws of motion of the capitalist mode of production. The tendency for the income-to-effort rate, or profit rate to fall is an unintended result of the pursuit of increasing individual profitability. As the individual capitalist seeks higher profits by lowering costs and increasing income, they contribute to the macro phenomenon of declining profitability. This drive of the capitalist is rational from an individual point of view but is not from a macro-perspective. According to Marx, the falling profit rate will inevitably lead to the end of capitalism.

The process of the falling profit rate process arises as agents maximize their own return. The capitalist enters a line of production where the profit rate is greater than the average. As the capitalist enters that line of production, the supply increases and reduces prices. This leads consequently to a fall in the profit rate of that line of production. This is a negative feedback effect. The capitalist responds to the profit rate by entering or exiting markets according to their profit rates. Their actions affect that variable, creating a negative feedback result that equalizes the profit rate level in all sectors.

Classical Economic theory understands society as being divided into classes. These classes are defined by their role in the social reproduction of material life and mark the existence of different and heterogeneous agents. Agents have a trial-and-error method of decision-making, where the evolution of society is linked to institutional, environmental, and behavioral constraints. Classical theory is a historical theory, not a fixed-society description. The evolution of division of labor and profit rate levels are clear emergent phenomena that materialize at the system scale level born in the micro level.

Political Economy theorized in terms of the ceaseless and turbulent process of capital and labor migration depending on the profitability differentials. The resulting turbulent course of action was a consequence of the decentralized nature of capitalist production and the negative feedback system that ensures the gravitation process of prices. Marx described the probabilistic nature of political economy in a letter to Engels (Horowitz

& Horowitz 1968, p. 91, emphasis added):

> You know the tables representing prices, discount rates, etc., in the form of zigzags fluctuating up and down. I have tried repeatedly to compute these 'ups and downs' [the English expression is used by Marx] - for the purpose of business cycle analysis – as irregular curves and thus to calculate the principal laws of economic crises mathematically. *I still believe that the task can be accomplished on the basis of a critically sifted statistical material.*

Marx intentionally described social and economic phenomena in terms of how individual decisions would emerge as aggregate behavior. Marx pointed out how "fundamental determinations" of individual decision-making play out in the "aggregate or average behavior of a system". Those fundamental principles are often described by conservation principles, where a limited amount of resources is available and each individual gain must be accompanied by a similar size loss (Foley 2009, p. 25).

Both Neo-Ricardian and Marxian economists describe the profit rate as an aggregate behavior based on fundamental determinations. The tendency to equalize the profit rates focuses on the trajectory of the profit rate and does not imply that the profit rate is actually equalized. The tendency to equalize is a deterministic approach ('will equalize') which leads to a nondeterministic behavior ('is equalized'). The probabilistic approach provides a statistical disturbance to a deterministic model (Farjoun & Machover 1983). Such a statistical disturbance is apparent in the rate of profit. Empirical investigation exhibits much variation in the realized rate of profit of firms that gives rise to a probability distribution (Alfarano et al. 2012, Scharfenaker & Semieniuk 2017).

To consider the rate of profit, or other economic variables, as random allows us to use a probabilistic framework. As researchers, we can characterize such distributions and find inter-relations between them (Farjoun & Machover 1983). We move away from fix point solutions where the form of the obtained result (a Dirac Delta Function) reduces the 'fundamental determinations' of individual decision-making. Instead, we refocus on classical political economy reasoning. We concentrate on models in which the solution is a probability distribution.

5 A New Equilibrium

Statistical equilibrium models combine the ideas of entropy and probabilistic thinking. In this framework, the actions of agents are probabilistic and therefore not observable. The Quantal response model provides an avenue for researchers to analyze statistical equilibria with the underlying behavioral mechanism. We discuss how entropy is an important aspect of those models and its use in the statistical equilibrium literature.

The statistical equilibrium approach serves as a new research agenda for economists looking for more realistic models. Statistical equilibrium models are not one specific model, but a family of models born in Physics in the late 19th century by Boltzmann (1871), Maxwell (1860), and Gibbs (1902). Statistical equilibrium models incorporate how pertinent constraints influence observed distributional outcomes. This family of models does not want to and cannot misconstruct social systems as "social physics" as there are no fixed 'social laws' that individual social actors act as atoms from physical systems.

Statistical equilibrium models are capable of dealing with equilibrium in systems with many degrees of freedom, and uncertain and incomplete information. There are four important components: (1) statistical equilibrium models are inherent representations of complexity theory and (2) they avoid falling into the trap of using unrealistic mathematical assumptions. (3) The methodology shifts Economics from deterministic models towards probabilistic reasoning. (4) Lastly, it approximates Economics to other fields of science such as Physics which has transitioned from deterministic Newtonian models to Statistical Mechanics and other probabilistic approaches. Political Economy is an inherently probabilistic field, which anticipated many of the debates from complexity theory such as heterogeneous agents, evolutionary perspective, emergency, and bounded rationality. This makes Political Economy and Economics as a whole a predesignated field to be analyzed and modeled in a statistical equilibrium framework.

The statistical equilibrium methods imply equilibria in terms of a probability distribution and not a single moment of that distribution. The probability distribution can be found by including relevant institutional, environmental, and behavioral constraints that shape the distribution, described by a central moment and some variability around it. The statistical equilibrium agenda embraces the idea of economic outcomes being inherently statistical. None of the variability in the model comes from error terms exogenously introduced. On the contrary, the system's state variables themselves behave endogenously and probabilistically.

Economic variables are treated as random variables in the statistical equilibrium approach. By this, the variables do not assume a unique value, but multiple values with different probabilities. As the variables are inherently probabilistic, it is crucial to find a way to quantify the amount of uncertainty or surprise in an event. In the context of economics, entropy can be used to measure the average amount of information needed to represent an event drawn from a probability distribution for a random variable. This provides valuable awareness of the behavior of complex economic systems and gives the researcher the ability to provide a rigorous and statistically-based approach to understanding economic systems. The inference used in statistical equilibrium is born from digital communication. In the derivation of a measure of information, Shannon presented a quantity of uncertainty called entropy. Later, Jaynes (1957) recognized the generality of Shannon's result and its similarity with the statistical mechanics of assigning probabilities to the distribution of molecules' motions and states in systems with interacting particles, called many-body systems.

One way to address the probability of actions and states is Shannon's entropy (Shannon 1948). Shannon's entropy originates from the field of communications where the transmitted signals might be altered and the received message differs from the sent message. Based on Shannon's entropy it is possible to derive the most likely message based on the received signals. The procedure can be generalized to all forms of problems in the fields of statistical mechanics where probabilities are assigned to states and actions (Jaynes 1957).

Optimization is a fundamental concept in decision-making, where the goal is to find the best possible solution among many alternatives. This is mathematically framed with Lagrangian and Hamiltonian for static and dynamic variables. Economists are familiar with this method. They optimize the Marshallian demand for a good or costs for a firm. Both optimizations are under sub-conditions such as budget or technological constraints. Both maximization and minimization problems search for optimal solutions for decision-making problems, and they are solved using the Lagrange multipliers method (or Hamiltonian) because the existence of constraints enforces interior solutions. Statistical equilibrium models follow the same procedure: The setup of the problem is the search for a probability distribution for the state variables of the system. According to specific questions, the researcher uses specific constraints which appropriate to that scientific inquiry.

5.1 Entropy: Combinatorics

One of the most intuitive ways to derive the entropy equation is by counting events for a many-body problem type of system governed by complex interactions with a large number of degrees of freedom. In the case of the economy, we model the components[3] as the economic entities and their idiosyncratic characteristics: consumers with tastes, preferences, income, and education. Similar can be done with firms and their technology, capital stock, and employees. As many agents interact in a complex way which introduces many values in the final calculation of a volatile variable. With this large amount of degrees of freedom, we partition the description space ("coarse-graining"). Coarse-graining is used to simplify complex systems and make them more manageable for analysis and modeling. This reduces the number of degrees of freedom of the system by replacing the fine-grained description of the system with smoother versions of it.

By counting events, we create different bins in which the variable could be allocated. Let's look at the price of a commodity. The price can be anything between zero and infinite. We describe the realizations for the price by bins, associated with each realization. We count the number of observations that have a price between 1€and 1.25€, 1.25€and 1.50€, and so on. When plotting the price histogram (macrostate) we see the probability distribution for the individual prices and the probabilities associated with its occurrence. The specific macrostate does not tell us anything about the microstates of the system. Any price distribution (histogram) can be achieved in infinite ways, so we can not say which configuration has generated the specific distribution. Entropy helps to understand how many different price configurations can generate the macrostate observed in the particular histogram.

[3]This methodology is totally contrary to the atomistic approach of treating all the complexity of humankind as mere agents. That is, people react to an external stimulus as atoms would do under the same type of action. The Maximum Entropy program and Maximum Entropy Economics are neither adopting metaphors from physics nor any kind of methodological individualism.

By using the multiplicity, we count in how many ways the macrostate of the price distribution can be achieved. To generalize this counting mechanism, we use the multinomial coefficient. Expressed in a fraction, the numerator accounts for the permutation of the number of prices, and the denominator is the multiplication of the permutation of how many prices are in each bin. In a system with N elements, j states, and K bins we have, where states mean the price realization, and bins represent the price's range:

$$W = \binom{N}{n_K^j} = \frac{N!}{\left(n_1^j!\right)\left(n_2^j!\right)\dots\left(n_K^j!\right)} \tag{1}$$

Using Stirling's approximation, $\log(N!) \approx N log(N) - N$ for $N \gg 1$, and applying it to the multinomial coefficient (W) we have:

$$H = \log W = -N \sum_{k=1}^{K} p_k^j \log\left(p_k^j\right) \tag{2}$$

where $p_k^j = \frac{n_k^j}{N}$ or the frequency. The entropy function, H, is the degree of dispersion of the variable under study between the uniform (maximum entropy) and the degenerate distribution (minimum entropy). When $W = 1$ we have a degenerate distribution that puts all weight on one result, as all prices are in the same bin. If a commodity has a neoclassical equilibrium price p^*, then $H = \log(1) = 0$. A system with zero entropy represents the unattainable absolute zero, which is a violation of physical systems. In social sciences, this is similar to the probability of one state for a variable being 100%, while the probabilities for all other realizations are zero. Price divergence from the equilibrium price is therefore *by definition* not possible. The opposite occurs when we have a uniform distribution, where the prices are evenly spread out across the different bins or price ranges. In this case, $\left(n_1^j!\right) = \left(n_2^j!\right) = ... = \left(n_K^j!\right)$ and then $H = log(K)$.

We do not know all the information about the complex system comprised of multiple independent agents interacting. Entropy is a counting procedure to show the number of ways a particular distribution can be realized. Therefore, entropy is related to systems for which we have incomplete information. The lack of information, or "uncertainty about the system" is similar to the dispersion of the variables (Scharfenaker & Yang 2020).

5.2 Entropy: Information theory

Entropy can be derived as an informational concept as well. Shannon (1948) solved the problem of how to transmit information efficiently in a data communication system composed of three elements: a source of data, a communication channel, and a receiver. He presented the entropy rate, a quantity that measured a source's information production rate and also a measure of the information carrying capacity, or the communication channel capacity. For any given degree of noise contamination of a communication channel, it is possible to communicate discrete data nearly error-free up to a computable maximum rate through the channel. This is called the noisy-channel coding theorem or Shannon's theorem (MacKay 2003). The axiomatic derivation of entropy fulfills the conditions under which the problem could be solved:

1. Entropy, or the the average level of "surprise", or "uncertainty" inherent to the random variable's possible outcomes must be a continuous function of the probabilities,
2. For a random variable uniformly distributed, uncertainty must be a monotonically increasing function of the number of the outcomes of that random variable,
3. If the outcome can be decomposed into further components, then the uncertainty of the new system "should be the sum of the uncertainty of the old system plus the uncertainty of the new sub-systems weighted by its probability" (Scharfenaker & Yang 2020)[4]

Shannon proved that the only function that could satisfy the three conditions is the following:

$$H = -K \sum_{i=1}^{K} p_i \log (p_i) \tag{3}$$

Jaynes (1957) recognized the generality of Shannon's result and its similarity with the problem faced by Statistical Mechanics of assigning probabilities to quantum states. He challenged the frequentist interpretation to assign probabilities to events that may not repeat in reality. He advocates that a more appropriate interpretation of probabilities reflects a degree of belief in a hypothesis or a Bayesian interpretation of probability. It is possible as well to model with entropy-constrained methods where the understanding of probabilities does not rely on Bayesian interpretation of probabilities (Golan 2017). In the interpretation of information theory, entropy is a measure of our ignorance or uncertainty about the system. Shannon's entropy equation is understood as "state of knowledge in the sense common to Bayesian reasoning" (Scharfenaker & Yang 2020, p. 1583).

5.3 Statistical Equilibrium Models

When one optimizes Shannon's entropy as the objective function and adds theoretical constraints, the solution is a probability distribution. Maximizing entropy subject to a normalization constraint (the sum of probabilities equals one) produces a uniform distribution, where all outcomes are equally likely. When additional constraints are introduced, the distribution takes on specific forms or shapes due to the limitations imposed by these constraints. By including a first moment, such as the arithmetic mean, in the constraints, the maximum entropy problem solves for the exponential distribution. If the geometric mean is used, the resulting distribution follows a power law.

[4] An event with three outcomes has the following probabilities: $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{6}$. The entropy associated with this problem is $H(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$. If this problem is broken down into two equally likely choices with probability $\frac{1}{2}$, and the second choice occurs with probabilities $\frac{1}{3}$ and $\frac{2}{3}$, then its entropy is $H(\frac{1}{2}, \frac{1}{2}) + \frac{1}{2}H(\frac{1}{3}, \frac{2}{3})$, which is equal to the other entropy.

A normal distribution results from maximizing entropy subject to both a normalization constraint and constraints on the mean and second moment (variance). Therefore, when constraints are applied to complex systems they alter the maximum entropy distribution, illustrating the influence of information utilization in the process of inference. The major task of research is to figure out from available information what the relevant constraints are. The "constraints represent all of the information used that must be satisfied in the optimization process. They capture the rules that govern the system modeled" (Golan & Foley 2022). This is far from being an easy task, it is "the most important issue in setting up [constrained maximum entropy] problems is the specification of the constraints to reflect all of the information we have in a complete and consistent way"(Golan & Foley 2022).

Foley (1994) derives a statistical model to account for market exchanges considering all feasible Pareto-improving market transactions. He compares his model with the Walrasian competitive equilibrium theory and concludes that (1) all feasible Pareto-improving multilateral trades can occur with some probability in the statistical equilibrium, (2) equal agents will have different resulting payoffs as they trade in different price ratios, what Foley called "horizontal inequality" into market outcomes, and (3) statistical equilibrium goes in the direction but does not achieve Pareto-efficiency due to the existence of mutually advantageous trades. In conclusion, both unemployment and excess capacity are statistical equilibrium phenomena of the model. Those results arise because the statistical equilibrium does not require convexity of preferences or technology which is beneficial for a model not conjecturing unrealistic assumptions.

This approach is extended to a labor market that clears at entropy prices (Foley 1996). In this statistical equilibrium model, the equilibrium is achieved with a mean wage above the reservation wage. This makes the model compatible with involuntary unemployment. The compatibility of the statistical equilibrium model with unemployment eliminates the understanding of unemployment as an anomaly, present in Walrasian-Marshallian tradition. This model has strong assumptions such as no learning, experience, or memory process and is a single-run model.

Many other variables can be analyzed in a statistical equilibrium framework. Great effort has been put into the analysis of competition and how capital and power move between firms and sectors. The variables under consideration have been profit rates Tobin's q (Scharfenaker & dos Santos 2015), (Scharfenaker & Semieniuk 2017), stock returns (Citera 2021) or the markup (Weber 2023).

5.4 Quantal Response Statistical Equilibrium

The latest evolution of the application of entropy-constrained methods in Political Economy is the quantal response statistical equilibrium. The name of this class of models adds "quantal response" to the statistical equilibrium methods. Thereby the actions of the agents are not expressed in step functions around the threshold but rather probabilities of action which change with the realization of the relevant variable. So does the flow of capital stabilize the profit rate (Scharfenaker & Foley 2017). This is a direct conclusion of the notion of competition in Smith (1976). In the quantal response model, two mechanisms lead to this stabilization. First, capital owners are sensitive and react to profit rate differentials. They enter into sectors or trades with higher-than-average profit rates and exiting for lower-than-average profitability. Secondly, exiting a market increases profitability in that market while entering lowers it. The quantal response is conditional on the decisions of agents of entering and exiting markets or trades depending on profit rate levels.

Scharfenaker & Foley (2017) discuss how the impact of these actions dependent on profit rates, $f(a|x)$, and the impacts on profitability dependent on actions, $f(x|a)$. The two conditional distributions represent the interaction between profit rates and actions of entering and exiting. This formalizes the mutual dependency interaction between the quantal actions and the outcome. The statistical equilibrium is a joint distribution $f(a,x)$ of the two conditional probabilities, where $f(a,x) = f(a|x)f(x) = f(x|a)f(a)$. The model can be expressed either directly, by the joint distribution or indirectly, by marginal and conditional probabilities. To solve the system, the authors work with constrained entropy:

$$\begin{aligned} \max\{f(a|x) \geq 0\} \sum_a f(a|x)u(a,x) \\ \text{subject to} \quad \sum_a f(a|x) = 1 \\ -\sum_a f(a|x)\log(f(a|x)) \geq H_{\min} \end{aligned} \tag{4}$$

The optimization problem can be rewritten as a Lagrangian optimization problem, with the two Lagrangian multipliers T and λ such as:

$$\begin{aligned} L = -\sum_a f(a|x)u(a,x) - \lambda\left(\sum_a f(a|x) - 1\right) \\ +T\left(-\sum_a f(a|x)\log(f(a|x)) - H_{\min}\right) \end{aligned} \tag{5}$$

The Lagrange multiplier T is the behavior temperature that measures the attention or sensitivity of agents to differences in payoff. It limits the degree to which the agent will cluster around the highest payoff action. The lower T is, the more is the agent alerted to differences in the rate of profit. The outcome at the macro level will be closer to the unconstrained payoff-maximizing outcome (degenerate Dirac Delta distribution).

The duality of the constrained entropy optimization can be represented by the optimization of the expected payoff constrained by a minimum entropy level. The payoff-constrained entropy maximization is a general model when the assumptions are not known. This is a suitable approach for systems that change smoothly over time. The dual version of maximizing the expected payoff subject to a minimum entropy is a mathematical equivalent closely related to economic models. The entropy-constrained payoff maximization of the entropy-constrained quantal response model is a generalization of rational choice theory. In rational theory, the agents have well-defined payoff functions which they maximize by choosing a mixed strategy. This problem results in more frequent choices of a higher-payoff action. The entropy constraint ensures a level of randomness or unpredictability in the system. This can be particularly useful in situations where the decision-maker wants to avoid concentrating too much frequency on the highest-payoff action.

The sector profit rate shifted by a constant μ is a good approximation for the payoff. The payoff function is $u(a, x) = x - \mu$ leads to the quantal response functions:

$$f(enter|x) = \frac{1}{1 + e^{-(\frac{x-\mu}{T})}} \text{ and } f(exit|x) = 1 - f(enter|x) = \frac{e^{-(\frac{x-\mu}{T})}}{1 + e^{-(\frac{x-\mu}{T})}} \tag{6}$$

These functions are the logit quantal response or Gibb's distribution. For $T \gg 0$, $f(a|x)$ gets flatter which translates as a weak dependence between actions on profitability. This increases uncertainty and implies larger fluctuations in the aggregate profit rate. The opposite, $f(a|x) \approx 0$ strengthens the dependence of a on x. This reduces uncertainty and fluctuations in the profit rate. With lower impacts from entry-and-exit dynamics on profit rates $f(a|x)$, becomes a Heaviside step function. In this function, small deviations of x from μ result in a collateral response from agents in order to eliminate this deviation (see Figure 1). The asymmetry in the frequency distributions of the

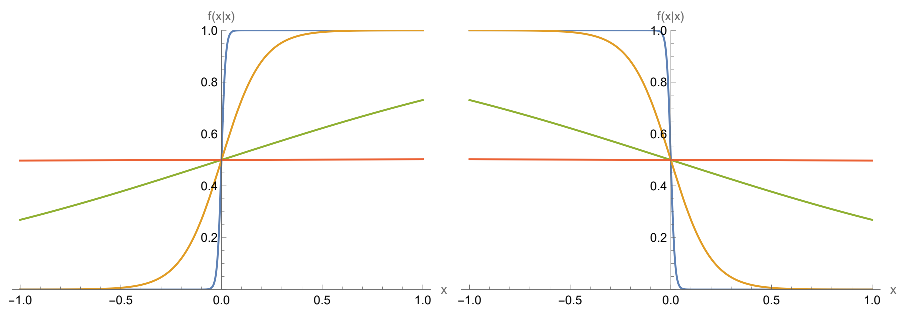

Figure 1: The logit quantal response function for different behavioral temperatures T. Blue: $T = 0.01$, Yellow: $T = 0.1$, Green: $T = 1$, Blue: $T = 100$.
Left: The probability of entering. Right: The probability of exiting.

rate of profit comes from unfulfilled expectations of the entropy-constrained behavior of the decision-makers (Scharfenaker 2020). When agents make wrong predictions or their expectations remain unfulfilled, the distribution of the outcome becomes asymmetric.

6 Conclusion

The statistical equilibrium framework is a new research agenda for economists looking for more realistic models of economic behavior. It takes into account the role of randomness and uncertainty in economic activity. This is done in a probabilistic way which does not rely on external shocks but purely on the probabilistic modelling of decisions and actions. This makes the statistical equilibrium framework a successor of the Classic understanding of economics when it was still called Political Economy.

We suggest their application to Classical Political Economy. To merge the statistical equilibrium methods with Political Economy arises from the recognition that Classical thinking was inherently statistical. Probabilistic models reduce unrealistic assumptions both theoretically and methodologically. When working with statistical equilibrium models, we emphasize how crucial it is to correctly set the constraints that will shape the distribution because of the role of non-linear interactions and feedback loops in economic activity.

The statistical equilibrium approach relies heavily on the concept of entropy. By providing a formal concept that includes uncertainty about the system and incomplete information in the modeling process, we can capture the dynamics and diversity of real-world economic systems. We show that quantal response models, as a subgroup of statistical equilibrium models, provide a realistic and nuanced understanding of economic behavior. Hence, statistical equilibrium models can be used to study properties from distribution, labor markets, goods markets, and other relevant aspect of economic systems. We show how these characteristics can help to explain phenomena such as innovation, adaptation, learning, coordination, cooperation, conflict, and emergence in economic activity.

By considering the complex reality characteristics and modeling the economy through the lenses of the statistical equilibrium approach, we suggest that the notion of equilibrium that arises from this methodology is much more appropriate to describe the economic reality. The emerging understanding of reality is more nuanced and less restrictive. It considers the equilibrium as the resulting probability distribution with its first moment and variability, instead of single moment equilibrium such as in fixed-point theorems.

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