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Stefan Ederer
Miriam Rehm

On the possibility of an ever-increasing wealth concentration: Pasinetti, dual, and anti-dual equilibria in a Post-Keynesian framework

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On the possibility of an ever-increasing wealth concentration:

Pasinetti, dual, and anti-dual equilibria in a Post-Keynesian framework

Stefan Ederer and Miriam Rehm

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Abstract

This paper develops a stock-flow consistent Post-Keynesian model in the Kalecki-Steindl tradition with endogenous wealth accumulation and distribution, which captures the key aspects of the Cambridge debate on (anti-)dual and Pasinetti equilibria. We find that a stable interior solution – that is, a Pasinetti equilibrium – is the most likely outcome, while the corner solutions of dual and anti-dual equilibria – both the euthanasia and the triumph of the rentier – are special cases of a standard Bhaduri-Marglin model. Endogenizing the profit share yields a two-dimensional dynamic system of the wealth concentration and the profit share, which is stable for a wide range of parameter values, as long as the concentration of wealth is not unrealistically low. An interior Pasinetti equilibrium thus remains the most likely outcome. However, for certain parameter combinations, the system may move onto an explosive trajectory with an ever-rising concentration of wealth and income in the hands of capitalists. Numerically illustrating the results of the analytical model shows that endogenizing the profit share leads to a more unequal wealth distribution, and a negative feedback effect between high wealth inequality, a high profit share, and growth.

1. Introduction

In the 1960s, Luigi Pasinetti was at the center of a debate between mainstream and heterodox economists on the theoretical foundations of the distribution of wealth and its determinants. In Pasinetti (1962), he argues that capitalists always own a certain share of the total wealth of an economy, and that this share is determined by their savings rate. Samuelson and Modigliani (1966) respond that if workers save a high enough proportion of their income, they will accumulate wealth faster than capitalists, who will eventually disappear as a social class. Pasinetti (1966, 1974), in return, dismisses this so-called "dual equilibrium" as empirically irrelevant. Finally, Darity (1981) shows the possibility of an "anti-dual" equilibrium, in which all wealth is concentrated in the hands of capitalists.

This debate was revived in the 2010s by Thomas Piketty's best-selling book "Capital in the 21st Century", albeit with reversed roles. Utilizing a neoclassical production function, Piketty (2014) argues that a rising ratio of wealth to income leads to a higher share of income for capitalists, which

dynamically feeds back into a rising concentration of wealth, leading essentially to an anti-dual equilibrium unless prevented by policy measures such as a wealth tax. Many Post-Keynesians, on the other hand, while appreciating Piketty's empirical work and sympathizing with his policy conclusions, found his theoretical framework lacking. Building on Pasinetti's (1962) "Cambridge equation", they argue that the distribution of wealth is stable in the long run (e.g., Galbraith 2014, Palley 2014, Taylor, 2014). Recently, an increasing number of contributions incorporate the dynamics of wealth distribution based on differential saving rates into economic models in the Kalecki-Steindl (Ederer and Rehm, 2020a), Structuralist (Taylor et al., 2015) or Classical-Marxist (Petach and Tavani, 2020) tradition. Some include further elements of the Cambridge debate relevant to the distribution of wealth, such as differential returns or capitalists receiving a certain share of labor income (Ederer and Rehm, 2020b), and, overall, support Pasinetti's position on the possibility of a stable distribution of wealth in the long run.

This paper contributes to these debates in two ways: First, inspired by Dutt (1990), we discuss the conditions for dual, anti-dual and Pasinetti equilibria in a Kalecki-Steindl model with an exogenous profit share, thereby summarizing the arguments in the literature in a consistent yet flexible model in the Bhaduri-Marglin (1990) tradition. Our model includes all the channels that determine the dynamics of wealth accumulation and distribution discussed in the literature, namely labor income going to capitalists, workers saving, differential returns on assets for workers and capitalists, and corporate savings, thereby placing the discussion within a broader analysis. Second, we discuss the dynamics between the distributions of functional income and wealth, and their consequences for the equilibrium wealth share if the assumption of an exogenous profit share is dropped. In doing so, we build on Taylor (2014), but place both the dynamics of the profit share and the wealth share in the medium run, and explicitly discuss their mutual feedback. We thus extend the Kalecki-Steindl literature on the distribution of wealth by allowing for a reinforcing dynamic between the distributions of income and wealth.

Our results show that the stable "Pasinetti equilibrium" of the wealth distribution is the most likely outcome both with an exogenous and with an endogenous profit share. In the first case, although both dual and anti-dual equilibria are theoretically possible, they are very special cases that exist only under certain, rather implausible parameter choices.

In the second case, while a stable equilibrium wealth share exists for plausible parameter values, the mutual feedback between the income and wealth distributions could potentially de-stabilize the dynamic system, leading to a trajectory of an ever-increasing inequality and thereby an anti-dual "Piketty equilibrium". However, both the wealth share and the profit share tend to self-stabilize, thus making a stable equilibrium a likely outcome if these forces are strong enough. We conclude, therefore, that while Piketty (2014) has brought to light an important channel through which inequality rises, that was neglected by Post-Keynesians, Pasinetti's (1962) insights into the dynamics of wealth distribution still carry the day.

The paper is structured as follows: Section 2 summarizes the relevant literature. Section 3 outlines the model. Section 4 discusses the conditions for dual, anti-dual and Pasinetti equilibria. Section 5 analyses the dynamic interaction between the profit share and wealth distribution. Section 6 concludes.

2. Literature

The seminal contribution of Pasinetti (1962) initiated a debate on the stability of the wealth distribution. By extending Kaldor's (1956) saving function to allow for both workers and capitalists to

accumulate wealth and consequently also to receive profit income,[1] Pasinetti (1962) finds that a stable wealth distribution is possible in the long run, and that it depends on the saving rates of workers and capitalists, as well as the functional income distribution. Samuelson and Modigliani (1966) and Meade (1966) respond by arguing that if workers' saving rate is high enough, they accumulate wealth faster than capitalists, so that the latter eventually vanish as a social class. Pasinetti (1966, 1974) responds that this so-called 'dual equilibrium' is empirically irrelevant. Later, Darity (1981) uncovers a third possibility – an 'anti-dual' equilibrium – in which all wealth is concentrated in the hands of capitalists.

This debate fruitfully linked to two other strands of literature, which focused on differential returns on the one hand, and corporate savings on the other hand. The former discussed the dynamics resulting from the assumption that workers receive an interest rate on their wealth ownership which is lower than the profit rate of capitalists. While Pasinetti (1962) still assumes these two rates to be equal, his seminal work (Pasinetti 1974, 1983) contributed to the discussion on differential returns following their conceptualization by Kahn (1959). Other work in this strand of literature includes Laing (1969), Balestra and Baranzini (1971), Harcourt (1972), Maneschi (1974), Moore (1974), Gupta (1977), and Fazi and Salvadori (1981). The reasons for differential returns across the wealth distribution might lie, among others, in more professional wealth management at higher wealth levels, the ability to take higher risk, or a higher likelihood of insider knowledge. Recent work provides empirical evidence for differential rates of return (Fagereng et al. 2020, Bach et al. 2020, Ederer et al. 2021, Petach and Tavani 2021, Bundesbank 2022). Differential returns also play a role in Piketty's (2014) argument for an ever-increasing wealth concentration, that is, an anti-dual equilibrium.

A second strand of the Post-Keynesian literature focused on retained earnings and thus saving by firms. The seminal work in this debate is Kaldor (1966), which defends the Kaldorian saving function in a response to Samuelson and Modigliani (1966) by arguing that retained earnings of firms are the reason why the saving rate out of profits is higher than the saving rate for wage income, given that the saving rate of firms is by definition equal to one. In a "neo-Pasinetti model", Kaldor includes firms' savings and thus shifts the analysis to a distinction between households and firms instead of workers and capitalists. Furthermore, the paper introduces a "valuation ratio", that is, the relation of the market value of shares to the reproduction cost of capital employed by firms, which acts as a mechanism equilibrating savings and investment; this concept was popularized as Tobin's (1969) 'q'. Chiang (1973), Moore (1974), and Darity (1981) generalize the analysis of the long-run equilibrium by distinguishing between workers, capitalists, and firms, all of whom save different fractions of their income. Darity (1981) explicitly models how the wealth accumulated by firms is distributed among socioeconomic classes by assuming that the wealth of firms is also indirectly owned by capitalists.

These two strands of literature analyse the economy in its long-run equilibrium, in which the capital stock increases at the natural growth rate by assumption. Therefore, they usually do not formulate a general macroeconomic model which represents the full dynamics of wealth accumulation and distribution. Recently, a small but growing literature (Dutt, 1990; Palley, 2012; Taylor et al., 2015; Ederer and Rehm, 2020a) includes wealth accumulation and distribution into Post-Keynesian models in the tradition of Kalecki (1971) and Steindl (1952). These models allow for mixed income sources for both workers and capitalists and yield endogenous stable wealth and income distributions. Some of these contributions show that wealth concentration is likely to rise empirically in Europe (Ederer and Rehm, 2020a) and the US (Kumar et al., 2018). Ederer and Rehm (2020b) show that a Post-Keynesian model predicts dynamics that resemble Piketty's empirical observations of a rising wealth to income ratio and a rising wealth concentration in the medium-run adjustment phase to the long-term equilibrium. Some of these contributions include some of the above-mentioned elements of the

[1] Kaldor proposed different saving propensities for wage and profit income but neglected that workers then accumulate wealth and thus receive profit income. A 'logical slip', in Pasinetti's (1962) words.

Cambridge debate relevant to the distribution of wealth, such as differential returns (Ederer and Rehm, 2020a, 2020b; Petach and Tavani, 2021) and the saving of firms (Palley, 2012; Taylor et al., 2015).

The most relevant contribution to this paper, which discusses conditions of Pasinetti, dual and anti-dual equilibria within a Kalecki-Steindl framework, is Dutt (1990). It includes firms' savings as a factor co-determining the distribution of wealth and finds that in a "neutral" case, in which the wealth of firms is distributed the same way as the wealth held by households in the form of savings or other assets, dual and Pasinetti equilibria are the only possible outcomes. However, despite its prescience, Dutt (1990) has two drawbacks: First, it does not take differential returns into account, and second, it uses a "stagnationist" Kalecki-Steindl model. While state of the art at the time, it has since been replaced in the Kaleckian literature by the seminal Bhaduri and Marglin (1990) model due to its advances in terms of generality.

As opposed to the Kaleckian tradition, structuralist models usually complement the demand side with an endogenous determination of the profit share. However, papers in the structuralist tradition typically focus on the cycles of growth and distribution, so-called "Goodwin cycles" (Barbosa-Filho and Taylor, 2006). The dynamics of the income distribution are thus placed in the short run. Those papers that include an endogenous wealth dynamic, including Taylor et al. (2015), Kumar et al. (2018), Taylor et al. (2019) usually do not analyze the feedback between the wealth and the income distribution. Taylor (2014) is the only paper that discusses the possibility of a Pasinetti equilibrium in such a framework, but retains the short-run dynamics of the profit share. Furthermore, demand in this model is unambiguously "profit-led" and it does not include differential returns or retained earnings of firms. Taylor (2014) finds that a dual equilibrium is only possible if the saving rates of workers and capitalists are equal. If capitalists save more, then a Pasinetti outcome is most likely. However, if the distribution of wealth has a strong impact on the profit share, then an anti-dual equilibrium becomes possible.

This paper thus adds to these literatures by, first, developing a model of wealth dynamics in the modern Kalecki-Steindl tradition, incorporating mixed income of workers and capitalists that incorporates differential rates of return, and by analyzing the conditions for dual, anti-dual and Pasinetti equilibria. Second, we endogenize the profit share to bridge the gap to the structuralist strand of the literature, while retaining the medium-term perspective of the Kalecki-Steindl approach.

3. Model

The model is a standard two-class, Post-Keynesian formulation in the tradition of Kalecki (1971) and Steindl (1952). Growth is endogenous and determined by the investment decisions of firms. Since Kaldor (1962) and Pasinetti (1962), we permit saving rates to differ between workers and capitalists. Building on Dutt (1990), Palley (2012, 2017), and Taylor et al. (2015), and closely following Ederer and Rehm (2020a, 2020b), the model features an endogenous accumulation and distribution of wealth. Both workers and capitalists receive labor and capital income, with the latter governed by differential rates of return on their wealth. Moreover, the model allows for corporate savings, which indirectly translate into the wealth of workers and capitalists via capital gains.

As usual in the Kalecki-Steindl literature, the profit share π is exogenous[2] and determined by the relative power of workers and capitalists in the labor and product markets. Total income (Y) is thus divided between total profits (R) and the wage bill (W):

$$R = \pi Y \tag{1}$$

[2] The assumption of a fixed profit share will be dropped in section 5.

$$W = (1 - \pi)Y \tag{2}$$

Retained earnings of firms (which are denoted by subscript *f*) are defined by the saving rate of firms η on total profits:

$$R_f = \eta R \tag{3}$$

Distributed profits are therefore $(1 - \eta)R$. Through their savings out of retained earnings, firms accumulate wealth (V_f) over time, which is their capital (K) minus loans (L) and equity (E):[3]

$$V_f = K - L - E. \tag{4}$$

In the spirit of López-Bernardo et al. (2016), we include a valuation ratio q, which is defined as the difference between firms' net capital (capital minus loans) over their equity: $q = \frac{K-L}{E}$. We assume a positive saving rate of firms and thus q to be larger than 1: firms cannot have negative valuation in the long run. Plugging q into equation (4) yields $V_f = (q - 1)E$, where the factor $q - 1$ represents capital gains, that is, the change in the valuation of equity, which is ultimately owned by households.

Households receive a share of profits which corresponds to their share in this profit-generating wealth – that is, equity – but with differential returns between workers and capitalists. These are based on the empirical observation that the composition of wealth differs between workers and capitalists (Ederer and Rehm, 2020a; Ederer et al., 2020; Petach and Tavani, 2020): workers (denoted by subscript w) hold a larger share of their wealth in low-yield asset classes such as bank deposits, whereas capitalists (subscript r) tend to own businesses, and thus benefit more from the compound interest. We therefore follow a simplified portfolio choice approach (Godley and Lavoie, 2007), and assume deposits to be non-interest bearing, and the shares of profit-generating assets for workers (γ_w) and capitalists (γ_r) to be constant.

Taking retained earnings into account implies that capital gains matter for wealth allocation for both workers and capitalists. We formulate this as the share of assets held in wealth generating assets (γ_r, γ_w) depending on the valuation ratio (q):

$$\gamma_w V_w = q E_w \tag{5}$$

$$\gamma_r V_r = q E_r \tag{6}$$

The disposable income of capitalists then amounts to their share (α) in total work income (W) plus their share in distributed profit income $((1 - \eta)R)$, with the latter being determined by their share in total wealth (z) and the part of their wealth held in profit-generating assets (γ_r), normalized by total wealth (that is, capitalists' and workers' wealth shares $(z, 1 - z)$ times their wealth shares held in profit-generating assets $(\gamma_r,\ \gamma_w)$). Workers' disposable income is formulated analogously:

$$Y_r = \alpha W + \frac{\gamma_r z}{\gamma_w(1 - z) + \gamma_r z}(1 - \eta)R \tag{7}$$

$$Y_w = (1 - \alpha)W + \frac{\gamma_w(1 - z)}{\gamma_w(1 - z) + \gamma_r z}(1 - \eta)R \tag{8}$$

Total consumption (C) out of disposable income yields differential saving rates of workers (s_w) and capitalists (s_r), with the assumption that $s_r > s_w$:

[3] For the balance sheet matrix and the transaction flow matrix of the model see Appendix A1.

$$C = (1 - s_w)Y_w + (1 - s_r)Y_r \tag{9}$$

Investment, that is the growth of the capital stock, is determined by animal spirits, capacity utilization (u), and the profit share (π). In the Bhaduri and Marglin (1990) tradition, this formulation allows for both wage-led and profit-led demand growth regimes:

$$I = (\beta_0 + \beta_1 u + \beta_2 \pi)K \tag{10}$$

The aggregate goods market is always in equilibrium, output equals demand. Since we abstract from all other sectors, total demand consists of consumption of households and investment of firms, so $Y = C + I$. To de-trend income, profits, and investment, we normalize them to the capital stock. This yields stable solutions for capacity utilization $u = \mathrm{Y}/K$, the profit rate $r = R/K = \pi u$, and the growth rate of the capital stock g$= I/K$.[4]

The only asset in the model is the wealth ultimately owned by households (V), which is equal to the capital stock (K):

$$V = K. \tag{11}$$

Over time, both capitalists and workers accumulate wealth until the wealth share adjusts to its long-term equilibrium. Taking the derivative of the wealth share $z = V_r/V$ with respect to time and rewriting the differential equation yields:

$$\dot{z} = \frac{\dot{V}_r}{V} - \frac{\dot{V}}{V}z = \left(\frac{\dot{V}_r}{V_r} - \frac{\dot{V}}{V}\right)z \tag{12}$$

This expression is a dynamic version of the famous Cambridge equation, since in its equilibrium ($\dot{z} = 0$) it boils down to Pasinetti's (1962) original expression $s_r r = g$ when capitalists do not receive any wage income, there are no differential returns for workers and capitalists, and firms do not save.[5] It implies that the wealth share is stable when capitalists save exactly the amount that corresponds to their share in the increase in total capital (Palley, 2012, 2017b; Taylor, 2014). Palley (2017b) thus terms the Cambridge equation an "ownership equilibrium condition" since capitalists must save just enough to maintain their ownership share.[6] The distribution of wealth stabilizes itself at its equilibrium since owning a higher (or lower) share of wealth leads to a lower (or higher, respectively) percentage increase in capitalists' wealth than in total wealth.

A detailed expression for equation (12)[7] yields

$$\dot{z} = Au \tag{13}$$

with A being a quadratic function of z in an inverse u-shape, as shown in Figure 1. Since u is always positive for the model to be economically meaningful, the sign of $\dot{z}$ depends uniquely on A, and setting $A = 0$ yields the long-run equilibria for z. The two corner solutions of the model are $z = 0$ (a euthanasia of the rentier) and $z = 1$ (a triumph of rentiers). As discussed in Section 2, the Cambridge debate dubbed these "dual equilibrium" (Samuelson and Modigliani, 1966) and "anti-dual equilibrium"

[4] For the short-run solution of the model, see Appendix A2.

[5] That is, $\alpha = 0, \gamma_w = \gamma_r = 1$, and $\eta = 0$.

[6] This is equivalent to saying that the wealth share z is stable if the (percentage) increase in capitalists' wealth is equal to the (percentage) increase in total wealth, see the right-hand side of equation (12).

[7] See Appendix for the details.

(Darity, 1981), respectively. A third option is a stable interior solution, a so-called "Pasinetti equilibrium" (Taylor 2014). We investigate the conditions for these equilibria next.

[Figure 1 about here]

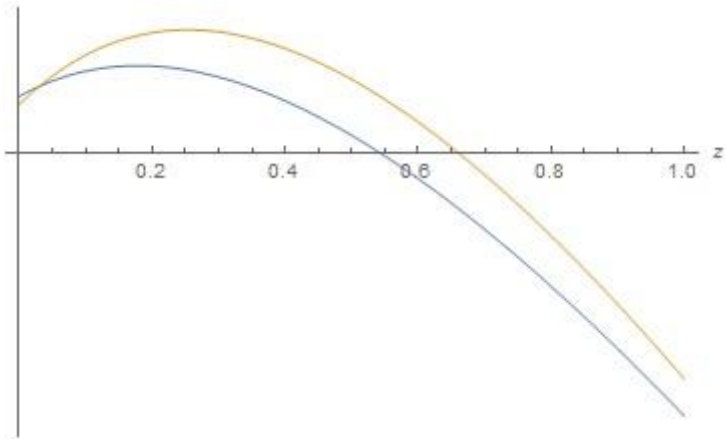
4. Dual, anti-dual, and Pasinetti equilibria

We first examine the possibility of a dual equilibrium, in which all wealth is concentrated in the hands of workers. If $z = 0$, then $A = s_r\alpha(1-\pi) > 0$ in equation (13), which implies an increasing share of wealth for capitalists. If the latter receive a certain share of wage income ($\alpha > 0$), they can always save and acquire wealth. Moreover, when the capitalists' wealth share is low, the percentage increase in their wealth is higher than the percentage increase in total wealth, which means that the wealth share of capitalists rises. A dual equilibrium is therefore not possible.

However, if capitalists do not receive any wage income ($\alpha = 0$), the wealth distribution does not change at $z = 0$, so that there is the possibility of a dual equilibrium. This equilibrium is nevertheless only stable under certain parameter conditions. For a reasonably small saving rate of workers, the slope of A is positive at a zero capitalists' wealth share, so that even the smallest deviation from the initial position $z = 0$ entails a rising wealth concentration and the dual equilibrium is therefore not stable. Only if the workers' saving rate is high, the slope of A becomes negative at $z = 0$, so that the dual equilibrium is the only stable outcome. The exact condition is

$$s_w > \frac{[s_r\gamma_r(1-\eta) + (\gamma_r - \gamma_w)\eta]\pi}{\gamma_w(1-\eta\pi)} \tag{14}$$

Figure 1:



Note: This figure shows the iso wealth curves $A = 0$ for $s_w = 0.07, s_r = 0.24, \gamma_w = 0.49, \gamma_r = 0.91, \alpha = 0.06, \eta = 0.1$; with $\pi = 0.29$ (blue line) and $\pi = 0.39$ (orange line).

Source: Own elaboration.

which is reduced to $s_w > s_r\pi$ if $\gamma_w = \gamma_r$ and $\eta = 0$, the case discussed by Pasinetti (1966) and Samuelson and Modigliani (1966).[8] Thus, the conditions for a dual equilibrium are that capitalists receive no income from wages and that the saving rate of workers is (perhaps implausibly) high. In this case, since workers get all the wage income, their savings are always higher than those of capitalists.

For reasonably small values of s_w, there is a stable and positive interior solution, a so-called "Pasinetti equilibrium" in which $0 < z < 1$. It exists if $A = 0$ for a certain wealth share $(0 < z < 1)$, or equivalently $A < 0$ for $z = 1$ (since $A > 0$ for $z = 0$ and A is an inversely quadratic function). At $z = 1$, $A = -s_w(1-\alpha)(1-\pi)$. Thus, if workers receive wage income and save, they will always hold a certain share of wealth. Only in the opposite case, when either workers do not save $(s_w = 0)$, all wages go to capitalists $(\alpha = 1)$, or all income goes to profits $(\pi = 1)$, workers' wealth share will be zero and the economy ends up in an anti-dual equilibrium, the triumph of the rentiers.[9]

If the saving rate of workers and capitalists is equal, that is, $s_w = s_r$, and both workers and capitalists have the same returns on their wealth $(\gamma_r = \gamma_w)$, equation (13) is reduced to a linear function which negatively depends on the wealth share z. In this case, the wealth distribution is governed by the distribution of wage income α and eventually stabilizes at its value. The economy thus ends up in a Pasinetti equilibrium. Only if capitalists do not receive wage income, the long-term equilibrium of the wealth share is zero, that is, the dual equilibrium.[10] Since in that case only workers receive wage income and both classes have the same saving rates (and returns), in the long-run wealth will be concentrated in the hand of workers. These assumptions are (implicitly) taken by Samuelson and Modigliani (1966), as pointed out by Taylor (2014).

To summarize, our analysis suggests that a stable interior solution – a Pasinetti equilibrium with a positive wealth share owned by both workers and capitalists – is the most likely outcome in a Kalecki-Steindl framework with plausible assumptions. Although both dual and anti-dual equilibria are theoretically possible, they are (very) special cases.

Another corollary of function A is that the Pasinetti equilibrium depends on the functional income distribution.[11] With a higher profit share, the income distribution shifts in favor of capitalists, since they hold a higher proportion of their wealth in profit-generating assets. A rising profit share thus increases the wealth concentration. Mathematically, $A(1) = -s_w(1-\alpha)(1-\pi)$ decreases (in absolute terms) with a rising profit share and the Pasinetti equilibrium shifts outwards. In the extreme case when $\pi = 1$, A is zero at $z = 1$, so that the economy eventually reaches anti-dual equilibrium and the rentiers triumph. This raises the possibility of an ever-increasing wealth concentration if the profit share is endogenous, to which we turn in the following section.

5. The mutually reinforcing dynamics of income and wealth distribution

In the previous section we assumed that the profit share is constant, as is common in Post-Keynesian models of the Kalecki-Steindl tradition. However, we also established that a higher profit share increases the concentration of wealth as it shifts the Pasinetti equilibrium towards a higher capitalists' wealth share. Furthermore, from the short-run solutions of the model in Appendix A2 one can see that

[8] Note that condition (14) is stronger than the original condition because the term on the right-hand side is augmented by differential returns and the saving rate of firms. The range of the workers' saving rate which leads to a Pasinetti equilibrium is therefore larger.

[9] This is in line with Dutt (1990), who showed that (without differential returns and no wage income going to capitalists), an anti-dual equilibrium is impossible even when the saving rate of firms is positive.

[10] Note that if $s_w = s_r$, condition (14) is fulfilled.

[11] See Ederer and Rehm (2020b).

an increase in the concentration of wealth has a dampening effect on capacity utilization. Supposing that the rate of capacity utilization and the rate of employment move in tandem, this implies a shift in the distribution of power in the labor market between workers and capitalists in favor of the latter, which, in a typical Post-Keynesian conflict-inflation setup, increases the profit share. With a higher profit share, however, the wealth concentration rises. This mutually reinforcing feedback between the distribution of income and the distribution of wealth is at the core of Piketty's (2014) argument, although it was developed within a neoclassical theoretical framework. In this section, we therefore drop the assumption of a constant profit share and systematically develop the above argument in our Post-Keynesian model in the Kalecki-Steindl tradition.

We follow Dutt (1994) in modelling the functional distribution of income as the outcome of conflicting claims of workers and firms mediated by a wage bargaining process. In such a model, wage inflation depends on the deviation of the actual wage share of workers from their target.[12] The target wage share follows Goodwin's (1967) interpretation of Marx' reserve army argument: workers increase their target when the employment rate is high, which, in our model, is approximated by capacity utilization. Conversely, firms aim at stabilizing the profit share and adjust prices when the profit share falls, passing rising wage costs on to consumers. The dynamics of the profit share then amount to

$$\dot{\pi} = \mu_1 B \pi \tag{15}$$

with $B = (\gamma_0 - \gamma_1 u - \pi)$, in which π is the profit share, u is capacity utilization, and the parameters $\mu_1, \gamma_0, \gamma_1$ follow from the wage-setting equation (equation (20) in Appendix A4). As usual in conflict-inflation models, the profit share stabilizes at a certain value, which depends negatively on the capacity utilization rate, and thus positively on the wealth share. Setting $B = 0$ yields this equilibrium for the profit share.

The dynamic equations for wealth share (equation (13)) and the profit share (equation (15)) form the two-dimensional differential equation system

$$\begin{pmatrix} \dot{z} \\ \dot{\pi} \end{pmatrix} = C \begin{pmatrix} z \\ \pi \end{pmatrix}, \tag{16}$$

with the Jacobian matrix

$$C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}.$$

The mutually reinforcing dynamic between the wealth share and the profit share is captured in the anti-diagonal elements of matrix *C*, c_{12} and c_{21}. For a detailed presentation of the elements of matrix *C*, see Appendix A5. A higher profit share increases the concentration of wealth (so that $c_{12} > 0$) for a wide range of parameter values, because it shifts the distribution of income in the favor of capitalists. Moreover, a higher capitalists' wealth share reduces capacity utilization and thus increases the profit share, due to firms being able to secure a higher share of total income ($c_{21} > 0$). Thus, under plausible assumptions for the parameter values of the model, both c_{12} and c_{21} are positive and there is a reinforcing dynamic between income and wealth concentration.

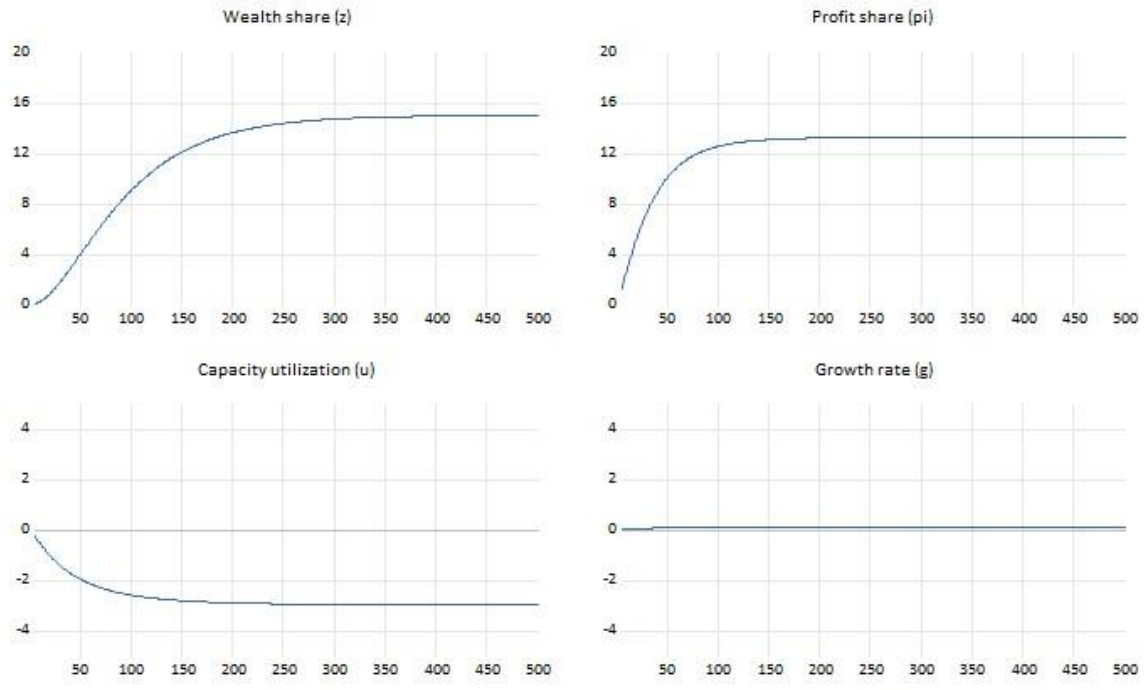
The sign of c_{11} follows from equation (13) and is negative if the wealth share is not too small. (See Appendix A6 for a numerical discussion of the stability of the dynamic system). All other things being equal, the wealth share stabilizes at its equilibrium because a positive (negative) deviation results in a lower (higher) percentage increase in capitalists' wealth than in total wealth, so that $c_{11} < 0$ (see

[12] We further assume that workers can stabilize the wage share by offsetting a certain fraction of price inflation and labour productivity growth (Flaschel, 2009). This simplifies the analytics without losing too much generality (Ederer and Rezai, 2021). For the equations for wage and price inflation, see Appendix A4.

section 4). The sign of c_{22} following from equation (15), on the other hand, is theoretically ambiguous and depends on the demand regime. In the profit-led case, a rising profit share increases capacity utilization, which in turn reduces the profit share through the shift of power in the labor market. In the wage-led case, a reinforcing effect is possible: A higher profit share dampens demand, which reduces employment and thus further increases the profit share. Nevertheless, in our conflict inflation model, any deviation from the profit share targets results in stabilizing the income distribution, with this mechanism being stronger than the destabilizing effect via the wage-led demand regime, so that $c_{22} < 0$.[13] Therefore, in our model, the two self-stabilizing forces counteract the mutually reinforcing dynamics between the wealth and the profit share.

The condition for a stable equilibrium is $c_{11}c_{22} - c_{12}c_{21} > 0$, in which case the dynamic system has two negative real Eigenvalues. It thus depends on the strength of the self-stabilizing mechanisms for both the wealth share and the profit share (that is, c_{11} and c_{22}) whether the dynamic system is stable. However, it depends on the parameter values whether the system follows an explosive trajectory or not. Numerical calculations in Appendix A6 show that the model usually yields stable income and wealth distributions. This is because changes in most of the parameters work in several, often opposite, directions. For example, a larger difference in saving rates between workers and capitalists makes capacity utilization more wage-led, thereby weakening the self-stabilizing tendency of the profit share. Thus, the value of c_{11} declines. On the other hand, it tends to make capacity utilization more sensitive to the wealth share (Ederer and Rehm, 2020b), increasing the mutually reinforcing dynamics between wealth and income distribution and raising c_{12}. The two effects therefore are of the opposite sign, which increases the likelihood that the dynamic system remains stable. Differential rates of return, the share of capitalists in wages (α), and the saving rate of firms (η) have similar effects.

Figure 2:



[13] This is inherent in all conflict-inflation models if $\gamma_1 \frac{du}{d\pi} < 1$.

Note: This figure shows the deviations from the constant profit share-baseline in percentage points if the profit share is endogenized. Parameter values: for $s_w = 0.07, s_r = 0.24, \gamma_w = 0.49, \gamma_r = 0.91, \alpha = 0.06, \eta = 0.1, \pi = 0.29$.

In summary, the dynamic system is likely stable for a wide range of parameter values. Endogenizing the profit share still leads to a Pasinetti equilibrium as the most likely outcome. However, for certain parameter combinations, the system may end up on an explosive trajectory with an ever-rising concentration of wealth and income in the hand of capitalists.

Another important implication of the above is that analyzing the distribution of wealth in the context of Post-Keynesian models most likely underestimates its concentration if the profit share is assumed to be constant. Endogenizing the profit share results in a higher share of capitalists in both wealth and income compared to the fixed profit share version of the model. The mutually reinforcing dynamics between these two therefore supports the concentration of wealth in the hands of capitalists. This process is not endless, however, but comes to an end at a (new and higher) Pasinetti equilibrium (Figure 2).

Furthermore, endogenizing the profit share also changes the effect of economic growth on the wealth distribution. Whereas the wealth share is impervious to changes in the growth rate with an exogenous profit share (Ederer and Rehm, 2020a), in the extended model a higher growth rate reduces both the wealth and the profit share. This is because higher growth improves the bargaining position of workers, resulting in a lower profit share. This in turn reduces the possibility for capitalists to accumulate wealth, thereby reducing their wealth share. The model thus reproduces both Pasinetti's and Piketty's conclusions in a Post-Keynesian macroeconomic framework.

6. Conclusion

This paper develops a stock-flow consistent Post-Keynesian model in the Kalecki-Steindl tradition with endogenous wealth accumulation and distribution, which captures the key aspects of the Cambridge debate on (anti-) dual and Pasinetti equilibria: mixed income sources for workers and capitalists, differential saving rates and differential rates of return, retained earnings (that is, saving by firms), as well as an endogenous profit share.

We find that the corner solutions of dual and anti-dual equilibria, that is, both the euthanasia and the triumph of the rentier, are special cases in a standard Post-Keynesian Bhaduri-Marglin model with an exogenous profit share, since they require workers not to save, and capitalists not to earn wage income (and workers' saving rate to be implausibly high). A stable interior solution – that is, a Pasinetti equilibrium – is therefore the most likely outcome.

Endogenizing the profit share in the model yields a two-dimensional dynamic system of the wealth concentration and the profit share. This system is stable for a wide range of parameter values, as long as the concentration of wealth is not unrealistically low. An interior Pasinetti equilibrium thus remains the most likely outcome in this Post-Keynesian model, even when the profit share is endogenous. However, for certain parameter combinations, the system may move onto an explosive trajectory with an ever-rising concentration of wealth and income in the hands of capitalists. Numerically illustrating the results of the analytical model shows that endogenizing the profit share leads to a more unequal wealth distribution, and a negative feedback effect between high wealth inequality, a high profit share, and growth.

Since the theoretical analysis of wealth inequality is an expanding field, several venues of future research remain open. A natural first step would be to empirically estimate parameter values and ranges while differentiating asset types (such as housing) in more detail, in order to move towards a more continuous distributional analysis. Extending the model by public wealth on the one hand, or adding an external sector with international stocks and flows of wealth would go further beyond the analysis presented here.

7. References

Barbosa-Filho, N. and Taylor, L. 2006. Distributive and Demand Cycles in the US Economy – A Structuralist Goodwin Model, Metroeconomica vol. 57, no. 3, 389–411.

Bhaduri, A. and Marglin, S. 1990. Unemployment and the Real Wage: The Economic Basis for Contesting Political Ideologies, Cambridge Journal of Economics vol. 14, no.4, 375–93.

Darity, W. A. 1981. The Simple Analytics of Neo-Ricardian Growth and Distribution, The American Economic Review, vol. 71, no. 5, 978–993.

Dutt, A. K. 1990. Growth, Distribution and Capital Ownership: Kalecki and Pasinetti Revisited. In: Dutta, B., Gangopadhyay, S., Mookherjee, D., Ray, D. Economic Theory and Policy. Essays in Honour of Dipak Banerjee. Bombay: Oxford University Press

Ederer, S., and M. Rehm. 2020a. Will Wealth Become More Concentrated in Europe? Evidence from a Calibrated Post-Keynesian Model. Cambridge Journal of Economics, 44(1), 55-72.

Ederer, S., and M. Rehm. 2020b. Making Sense of Piketty's "Fundamental Laws in a Post-Keynesian Framework, Review of Keynesian Economics 8(2), 195-219.

Ederer, S., Mayerhofer, M. and M. Rehm. 2021. Rich and ever richer: Differential returns across socio-economic groups, Journal of Post-Keynesian Economics.

Flaschel, P. 2009. Topics in Classical Micro- and Macroeconomics. Berlin: Springer.

Godley, W., and Lavoie, M. 2007. Monetary Economics. London: Palgrave.

Goodwin, R., 1967. A growth cycle, in: Feinstein, C. (ed.). Socialism, capitalism, and economic growth. Cambridge, UK: Cambridge University Press.

Kaldor, N. 1955. Alternative Theories of Distribution, The Review of Economic Studies vol. 23, no.2, 83–100.

Kaldor, N. 1966. Marginal Productivity and the Macro-Economic Theories of Distribution: Comment on Samuelson and Modigliani, Review of Economic Studies 33(4), 309-319.

Kalecki, M. 1971. Selected Essays on the Dynamics of the Capital Economy, Cambridge, UK: Cambridge University Press

Kumar, R., Schoder, C., Radpour, S. 2018. Demand driven growth and capital distribution in a two class model with applications to the United States, Structural Change and Economic Dynamics 47, 1-8.

Lavoie, M. 1998. The Neo-Pasinetti Theorem in Cambridge and Kaleckian Models of Growth and Distribution, Eastern Economic Journal, 24 (4), 417-434.

López-Bernardo, J., Stockhammer, E., López-Martínez, F. 2016. A Post-Keynesian Theory for Tobin'q in a Stock-Flow Consistent Framework, Journal of Post Keynesian Economics vol. 39, no. 2, 256-285

Meade, J.E. 1966. The outcome of the Pasinetti-Process: A Note, Economic Journal 76(301), 161-165.

Moore, B. J. 1974. The Pasinetti Paradox Revisited, The Review of Economic Studies, vol. 41, no. 2, 297-299

Palley, T. 2012. Wealth and wealth distribution in the neo-Kaleckian growth model, Journal of Post Keynesian Economics vol. 34, no. 3, 453–474

Palley, T. 2017. Wage- vs. profit-led growth: the role of the distribution of wages in determining regime character, Cambridge Journal of Economics vol. 41, no. 1, 49–61

Palley, T. 2017b. Inequality and growth in neo-Kaleckian and Cambridge growth theory, Review of Keynesian Economics vol.5, no. 2, 146–69

Pasinetti, L. 1962. Income Distribution and Rate of Profit in Relation to the Rate of Economic Growth, Review of Economic Studies vol. 29 no. 4, 267–279

Pasinetti, L. 1966. Paradoxes in Capital Theory: A Symposium: Changes in the Rate of Profit and Switches of Techniques, Quarterly Journal of Economics vol. 80, no. 4, 503-517.

Pasinetti, L. 1974. Growth and Income Distribution - Essays in Economic Theory, Cambridge: Cambridge University Press

Petach, L. and Tavani, D. 2018. Income Shares, Secular Stagnation, and the Long-Run Distribution of Wealth, http://dx.doi.org/10.2139/ssrn.3198311

Piketty, T. 2014. Capital in the twenty-first century, Cambridge, Mass.: Harvard University Press.

Samuelson, P. A., and F. Modigliani. 1966. “The Pasinetti Paradox in Neoclassical and more General Models.” *The Review of Economic Studies* 33 (4): 269–301.

Steindl, J. 1952. Maturity and Stagnation in American Capitalism. New York: Monthly Review Press.

Taylor, L. 2014. The Triumph of the Rentier? Thomas Piketty vs. Luigi Pasinetti and John Maynard Keynes, International Journal of Political Economy, vol. 43, no. 3, 4-17

Taylor, L., Ömer, Ö., Rezai, A. 2015. Wealth Concentration, Income Distribution, and Alternatives for the USA, INET Working Paper No. 17, https://www.ineteconomics.org/uploads/papers/WP17-Lance-Taylor-Income-dist-wealth-concentration-0915.pdf

Taylor, L., Foley, D.K., Rezai, A. 2019. Demand drives growth all the way: Goodwin, Kaldor, Pasinetti and the Steady State, Cambridge Journal of Economics, vol. 43, no. 5, 1333-1352.

Zamparelli, L. 2017. Wealth Distribution, Elasticity of Substitution, and Piketty: An 'Anti-Dual' Pasinetti Economy, Metroeconomica 68 (4), 927-946

A. Appendix

A.1 Balance-sheet matrix

Table A1 shows the financial balances of the economy. The economy consists of households (workers and capitalists), firms, and a very basic banking sector. Households own firm equities and bank deposits, which banks loan to firms. Firms' wealth is made up of their capital minus equities and loans. Total capital in the economy equals total wealth ownership, and the financial balances of households', capitalists', and firms' wealth, as well as banks' balance sheets, sum to zero.

Table A1: Balance sheet matrix of the endogenous wealth distribution model with capital gains

	Households				
	Workers	Capitalists	Firms	Banks	Total
Capital			$+K$		$+K$
Equity	$+E_w$	$+E_r$	$-E$		0
Deposits	$+D_w$	$+D_r$		$-D$	0
Loans			$-L$	$+L$	0
Wealth	$-V_w$	$-V_r$	$-V_f$		$-V$
Total	0	0	0	0	0

Source: Own presentation.

Table A2 shows the transactions in the economy. Households (workers and capitalists) receive wages paid by firms (W) as well as distributed profits $(R_{w,r})$. Firms retain a part of profits (R_f). Households' consumption expenditures (C) flow to firms. Firms invest (I), financed by retained earnings, equity inflows (ΔE) and new loans (ΔL). Households finance new equity in firms and deposit their savings in banks. Loans flow from banks to firms. Since the model is stock-flow consistent, all rows and columns sum to zero.

Table A2: Stocks and flows of the endogenous wealth distribution model with capital gains

	Households		Firms		Banks	
	Workers	Capitalists	Current	Capital	Capital	Total
Consumption	$-C_w$	$-C_r$	$+C$			0
Investment			$+I$	$-I$		0
Wages	$+W_w$	$+W_r$	$-W$			0
Profits	$+R_w$	$+R_r$	$-R$	$+R_f$		0
Equity	$-\Delta E_w$	$-\Delta E_r$		$+\Delta E$		0
Deposits	$-\Delta D_w$	$-\Delta D_r$			$+\Delta D$	0
Loans				$+\Delta L$	$-\Delta L$	0
Total	0	0	0	0	0	0

Source: Own presentation.

A.2 Short-run dynamics

The short-run solution of the model assumes the wealth shares to be constant since wealth only accumulates over a longer time horizon. Capacity utilization is assumed to adjust, so that the equilibrium in the aggregate goods market (that is, the IS-condition) is fulfilled for any wealth share of capitalists (z). The short-run solution for capacity utilization thus depends on the distribution of wealth which co-determines the aggregate saving rate s:

$$u^* = \frac{\beta_0 + \beta_2 \pi}{s - \beta_1} \tag{17}$$

$$\begin{aligned} s = s_w \left[(1-\alpha)(1-\pi) + \frac{\gamma_w (1-z)}{\gamma_w + (\gamma_r - \gamma_w) z} (1-\eta)\pi \right] \\ + s_r \left[\alpha (1-\pi) + \frac{\gamma_r z}{\gamma_w + (\gamma_r - \gamma_w) z} (1-\eta)\pi \right] + \eta\pi \end{aligned} \tag{18}$$

Capacity utilization u is positive if the (extended) Keynesian stability condition $s - \beta_1$ holds, which is commonly assumed in Post-Keynesian models in the Kalecki-Steindl tradition (Dutt 1990). Since the saving rate rises when the wealth share increases, capacity utilization falls (Ederer and Rehm, 2020a). The reason is that a higher wealth share transfers profit income to capitalists, which depresses total consumption due to their higher saving rate and their higher returns on wealth. Furthermore, since the growth rate g positively depends on capacity utilization, a rise in the wealth share unambiguously depresses growth.

From the definitions of the profit rate and the growth rate ($r = \pi u$ and $g = su$, respectively), it follows that the former is more sensitive to the wealth share than the latter, because the saving rate increases when the wealth concentration rises. This is the reason why the profit rate decreases faster than the growth rate when the wealth share rises, and the two variables will eventually fulfil the Cambridge equation.

A.3 Long-run dynamics

$$\begin{aligned} A = s_r \alpha (1-\pi) - [s_w (1-\alpha) + s_r \alpha](1-\pi) z \\ + \left[\frac{(s_r \gamma_r - s_w \gamma_w)(1-\eta)\pi + (\gamma_r - \gamma_w)\eta\pi}{\gamma_w (1-z) + \gamma_r z} \right] (1-z) z \end{aligned} \tag{19}$$

A.4 Conflict-inflation model

With defining the target profit share as $\pi_{Tw} = 1 - \Omega_{Tw} = \gamma_0 - \gamma_1 u$, and the profit share given by the identity $\pi = 1 - \Omega$, wages grow by

$$\hat{w} = \mu_1(\Omega_{Tw} - \Omega) + \hat{p} + \hat{a} = \mu_1[\pi - \gamma_0 + \gamma_1 u] + \hat{p} + \hat{a}, \tag{20}$$

For simplicity, we assume a constant target profit share $\pi_{Tr} = \delta_0$:

$$\hat{p} = \tau(\pi_{Tr} - \pi) = \tau(\delta_0 - \pi). \tag{21}$$

Taking time derivatives of the profit share $\pi = \frac{wL}{pY}$, we get

$$\dot{\pi} = -(\hat{w} - \hat{p} - \hat{a}) = [\mu_1(\gamma_0 - \gamma_1 u - \pi)]\pi = \mu_1 B\pi \tag{22}$$

with $B = (\gamma_0 - \gamma_1 u - \pi)$. Setting $B = 0$ yields the long-term equilibrium for the profit share.

A.5 Jacobian of the dynamic system

The elements of the Jacobian of the differential equation system around the long-term equilibria for z and π are:[14]

$$\begin{aligned} c_{11} &= \frac{\partial A}{\partial z} \\ &= -[s_w + (s_r - s_w)\alpha](1 - \pi) \\ &- \frac{[(s_r\gamma_r - s_w\gamma_w)(1 - \eta) + (\gamma_r - \gamma_w)\eta][\gamma_w(2z - 1) + (\gamma_r - \gamma_w)z^2]\pi}{[\gamma_w + (\gamma_r - \gamma_w)z]^2} \end{aligned} \tag{23}$$

$$\begin{aligned} c_{12} = \frac{\partial A}{\partial \pi} &= s_w(1 - \alpha)z - s_r\alpha(1 - z) \\ &+ \frac{[(s_r\gamma_r - s_w\gamma_w)(1 - \eta) + (\gamma_r - \gamma_w)\eta]z(1 - z)}{\gamma_w + (\gamma_r - \gamma_w)z} \end{aligned} \tag{24}$$

$$c_{21} = \frac{\partial B}{\partial z} = -\gamma_1 \frac{\partial u}{\partial z} \tag{25}$$

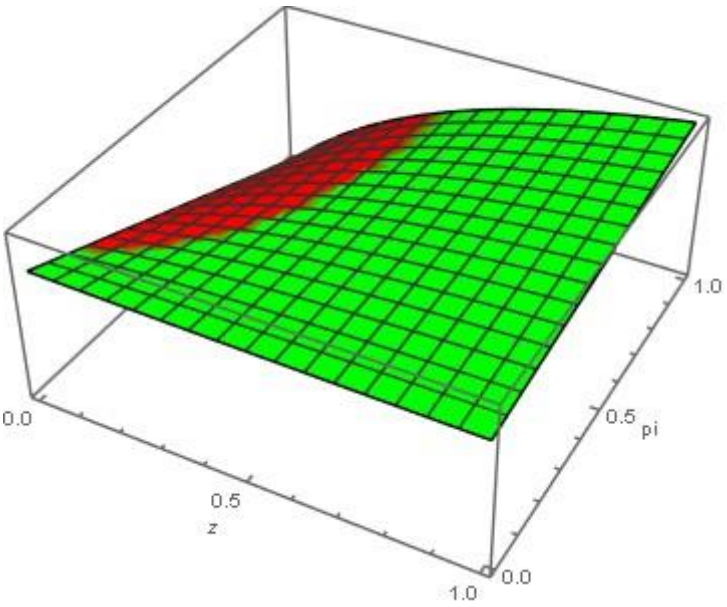
$$c_{22} = \frac{\partial B}{\partial \pi} = -\gamma_1 \frac{\partial u}{\partial \pi} - 1 \tag{26}$$

[14] Recall that we obtain the equilibria by setting $A = 0$ and $B = 0$.

A.6 Numerical discussion of the stability of the dynamic system

The condition for a stable equilibrium is $c_{11}c_{22} - c_{12}c_{21} > 0$, in which case the dynamic system has two negative real Eigenvalues (see section 5). Figure A1 shows this condition is fulfilled for sufficiently large values of the wealth share and the profit share. For low values of these two variables, however, the condition for stability is not fulfilled (that is, the red area in Figure A1) and the dynamic system is unstable, which results in an increase of both the profit share and the wealth share, so that eventually they move into a range where the system is stable (that is, the green area in Figure A1). Moreover, the extent of the red area does not change much when the parameters are varied, because the effect of these changes on the entries in matrix C often work in the opposite directions (see section 5).

Figure A1:



Note: The figure shows the stability condition $c_{11}c_{22} - c_{12}c_{21}$ as a function of π and z. Positive values (stability) are displayed in green, negative (instability) in red. Parameters: $s_w = 0.07, s_r = 0.24, \gamma_w = 0.49, \gamma_r = 0.91, \alpha = 0.06, \eta = 0.1, \beta_0 = 0.01, \beta_1 = 0.01, \beta_2 = 0.01, \gamma_0 = 0.5, \gamma_1 = 0.5$).

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ifso institute for socio-economics

Institute for Socio-Economics
University of Duisburg-Essen

Lotharstr. 65
47057 Duisburg
Germany

uni-due.de/soziooekonomie
wp.ifso@uni-due.de